Theoretical Basis of the Relational Model

Relational model, keys, integrity constraints, and transformation from ER

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Relational Model Basics

Basic Structure of the Relational Model

- In practice, **relations** are often compared to **tables** (e.g., in spreadsheets).
- The relational model is a **logical-level** data model: it describes *what* is stored and the allowed structure.
- We use a relation name and a set of **attributes** (columns); the table contains **tuples** (rows).

Example: a Relation as a Table (SUPPLIER)

- In a relational database all information of the logical database design is organized as a set of tables (relations).
- Here is an example table:

SupplierID	Name	City	FoundedYear
31	Worchware	Kaeli	2003
42	Fycbow	Shimo–carnia	1993
33	Myjice	Yarghe–pfuhar	2008
54	Bivoke	Lucan	1986
25	Vakoj	Idgipurn	2000

Relations can often be *thought of* as spreadsheet tables (but the relational model has strict rules and additional elements such as keys and schemata).

Core Definitions (high level)

- A **relation** R can be seen as a pair $\langle H, r \rangle$:
  - **Header** H: a set of attribute names (the "column names").
  - **Body** r: a set of tuples (the "rows").
- An **attribute** is an attribute name together with a domain of values.
- A **tuple** is an ordered collection of *attribute–value* pairs, one value per attribute in H.
- A **relational database** is a set of relations plus integrity constraints.

Auxiliary Terms

- **Relation schema** (or structure): the relation name plus its header.
- **Relation body**: the set of tuples in the relation.
- **Cardinality of a relation**: number of tuples (rows).
- **Cardinality of an attribute**: number of distinct values appearing in a column.

Worked Example: DELIVERY Relation

- Schema:

$$\texttt{DELIVERY}(\texttt{supplier\_no}, \texttt{part\_no}, \texttt{project\_no}, \texttt{quantity})$$

- Header is the attribute set $\{\texttt{supplier\_no}, \texttt{part\_no}, \texttt{project\_no}, \texttt{quantity}\}$.
- If the table has 5 rows, then the **cardinality of the relation** is 5.
- Example tuple (omitting attribute names):

$$(1,\ 2,\ \texttt{p123},\ 17)$$

- If the values in `quantity` are $\{17, 23, 9, 4, 12\}$, then the **attribute cardinality** of `quantity` is 5.

Special Value NULL vs. Quantity 0 in DELIVERY

- In a relation, an attribute value can be **known** (e.g., 17, 0) or **unknown / not provided** (NULL).
- NULL **in** quantity means: the quantity is *not defined / missing data*.
  - Example: the shipment record exists, but the delivered amount was not recorded.
  - NULL is *not* a number, and it is *not* equal to 0.
- **0 in** quantity means: the quantity is *exactly zero*.
  - Example: **"Loppu"** / *zero products available* is a precise statement: quantity $= 0$.
- Contrast with two tuples:

$$(1,\ 2,\ \texttt{p123},\ \texttt{NULL}) \quad \text{vs.} \quad (1,\ 2,\ \texttt{p123},\ 0)$$

- Consequence: treating NULL as 0 changes meaning and can corrupt results (e.g., sums, averages, and comparisons).

Revision (Quick Check)

Mark each statement as True/False.

1. The relation name is `DELIVERY`.
2. `supplier_no` and `part_no` are names of the relation's attributes.
3. The relation header is `supplier_no`, `part_no`, `project_no`, `quantity`.
4. The degree of the relation is five.
5. The cardinality of the relation is five.
6. In the relation, the cardinality of the attribute `part_no` is 4.
7. No null values appear in the relation.

Core Properties of Relations

According to the relational model, a relation should satisfy:

1. Every tuple has the same number of values (one per attribute; no "missing columns").
2. The order of tuples does **not** matter.
3. Tuples are **distinct** (no two identical rows).
4. The order of attributes in the header does **not** matter (names connect columns to meaning).
5. Attribute names should make the meaning clear (good naming matters).

Example of a Bad "Relation" (why rules matter)

- A "Top 5 suppliers" table can break relational rules if:
  1. some rows have missing values (null where not intended),
  2. row position is used as a rank (order would then matter),
  3. identical rows exist (duplicates),
  4. two columns share the same name (ambiguous),
  5. a column name is unclear (meaning not defined).
- Fixes: add an explicit `rank` attribute, enforce keys, rename columns, and define domains.

Keys and Integrity Constraints

Integrity Constraints

- Integrity constraints describe **allowed states** of a database.
- Key ideas introduced here:
  - **Primary keys** (uniquely identify tuples)
  - **Foreign keys** (references between relations)
  - Additional constraints (e.g., range constraints, business rules)

Primary Key and Candidate Key

- Every tuple should be **identifiable** (distinguishable) from all others.
- A **candidate key** is an attribute set that:
  - is **unique** (no two tuples share the same key value),
  - is **minimal** (removing any attribute destroys uniqueness).
- A **primary key** (PK) is one chosen candidate key (often underlined in schemas).
- Not always possible to choose a "perfect" key: stability and practicality matter.

Example: Candidate Keys in STUDENT

Consider a student registry relation:

$$\texttt{STUDENT}(\underline{\texttt{std\_no}},\ \texttt{ssn},\ \texttt{first\_name},\ \texttt{last\_name},\ \texttt{postal\_code},\ \texttt{city})$$

- Candidate keys in the domain:
  - $\{\texttt{std\_no}\}$ (university internal student number)
  - $\{\texttt{ssn}\}$ (social security number) *if stable and unique in your context*
- Not candidate keys (examples):
  - $\{\texttt{std\_no}, \texttt{ssn}\}$ is not minimal.
  - $\{\texttt{first\_name}, \texttt{last\_name}, \texttt{postal\_code}\}$ is not guaranteed unique.

Choosing Good Primary Keys (rules of thumb)

- Prefer keys that are:
  - stable (do not change),
  - easy to manage and index,
  - minimal (few attributes),
  - meaningful *enough* or safely artificial. *always defined*: should never be NULL
- Often, a synthetic identifier is used (e.g., sequential id, UUID, NanoID, Snowflake ID).

Foreign Key (FK)

- A **foreign key** (FK) in relation R is an attribute set whose values must come from a key of some relation S.
- Purpose: represent references and enforce **referential integrity**.
- Notation example:

$$R(\ldots, a, \ldots), \quad S(\underline{a}, \ldots), \quad R.a \to S.a$$

- A FK does *not* need to be unique; many tuples may reference the same target tuple.

Example Schema with PKs and FKs

- Primary keys are underlined, foreign keys indicated by arrows.
- Notes:
  - Attributes of a PK may also act as FKs.
  - FKs may be null (depending on constraints), unlike PKs.
  - If the referenced tuple is deleted/updated, the DBMS must handle the impact (restrict, set null, cascade).

Example (illustrative style):

SUPPLIER(supplier_no, name, city)
PART(part_no, name)
PROJECT(project_no, name, budget)
DELIVERY(supplier_no,part_no,project_no, quantity)

DELIVERY.supplier_no → SUPPLIER.supplier_no
DELIVERY.part_no → PART.part_no
DELIVERY.project_no → PROJECT.project_no

Revision + Other Integrity Constraints

Quick check (True/False):

1. A relation has one or more primary keys.
2. A relation can have more than one candidate key.
3. A tuple consists of attribute names and values.
4. A relation's foreign key refers to an attribute set (1..n attributes) of some other relation.

Examples of other constraints:

- Salary must not exceed supervisor's salary.
- Only one person who is also employed by the department can be a department manager.
- A phone number must start with +358.
- An attribute value cannot be null (NOT NULL).

Allowed Data Types and Normalization (preview)

- Attribute values are typically atomic (numbers, strings, dates, ...).
- If a relation violates modeling requirements, it may be **unnormalized**.
- It is often possible to fix unnormalized relations by **normalizing** them (later topic).
- Different data models and DBMSs support different data types (covered in the SQL part).

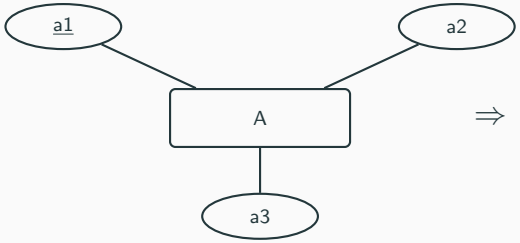
Transformation: From ER to Relations

Transformation (Conceptual → Relational)

- A conceptual model (e.g., ER diagram) captures domain concepts and relationships.
- The relational model has only **relations**, **primary keys**, **foreign keys**, and constraints.
- There is no direct 1:1 correspondence between ER constructs and relations.
- **Transformation** is the rule-based conversion of an ER diagram into a relational schema.
- Rules here are adapted from standard database textbooks (e.g., Elmasri & Navathe).

Rule: Strong Entity Set

Create one relation for each strong entity set.
Include all ordinary (simple) attributes.
The primary key is formed from the key attribute(s).



$\Rightarrow$ $A(\underline{a1}, a2, a3)$

Rules: Relationships with different Cardinality Constraints

- Relationship (with cardinality constraints). How many times min..max can/must an entity participate in the relationship set?
- Use **min** (0 or 1) and **max** (1 or N) near each side.

STUDENT(StudID, ...),COURSE(CourseID, ...), EnrolledIn(StudID, CourseID),
FK: EnrolledIn.StudId → STUDENT.StudId, EnrolledIn.CourseId → STUDENT.CourseId

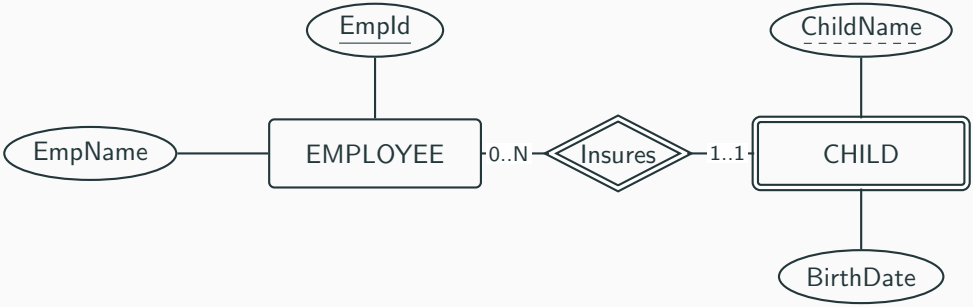
PERSON(PersonId, ...), CAR(CarID, PersonId, ...), FK: CAR.PersonId → PERSON.PersonId

Notably only two relations are used.

PLAYER(PlayerId,...), TEAM(TeamId,...), playsIn(PlayerId,..., TeamId),
FK: PlaysIn.PlayerId → PLAYER.PlayerId

Rule: Weak Entity Set

- Recap: A **weak entity** cannot be uniquely identified by its own attributes alone.
- It is identified through an **identifying relationship** with a strong entity.
- Rule of thumb: max cardinality of weak entity in identifying relationship is often 1.



Relational schema

`EMPLOYEE(EmpId, EmpName)`

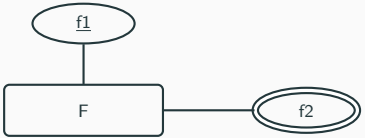
`CHILD(EmpId, ChildName, BirthDate)`

`CHILD.EmpId` $\rightarrow$ `EMPLOYEE.EmpId`

Rule: Transforming Attributes

- **Rule (Derived attributes)**: discard derived attributes (compute when needed).
- **Rule (Composite attributes)**: replace a composite attribute by its components; discard the composite node.
- **Rule (Multivalued attributes)**: create a separate "attribute relation" containing:
  - the key of the owner entity set, and
  - the multivalued attribute

  and make the owner key a foreign key.



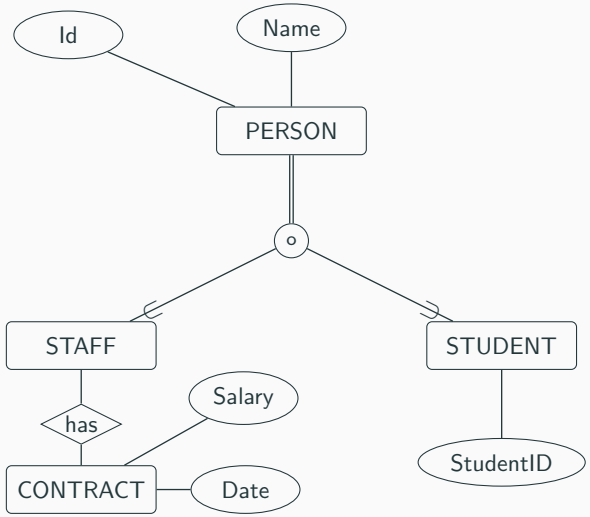
$$\Rightarrow \begin{array}{l} F(\underline{f1}) \\ F2(\underline{f1}, f2) \\ F2.f1 \rightarrow F.f1 \end{array}$$

Note on Cardinalities (in these rules)

- In the transformation rules below, cardinalities refer to **maximum** cardinalities.
- A 0:1 relationship means max participation of an entity in a relationship is 1 (e.g., "a person can be in at most one active marriage").
- A 1:1 relationship means exact cardinality is 1 (e.g., "a car has exactly one registration").
- In the relational model, **minimum** cardinality is generally not expressible in the standard ER model; additional integrity constraints are needed (we can encode them later, e.g., in SQL).

Recap: Abstraction Structures (EER)

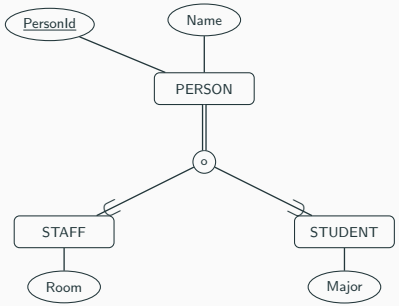
- Use **E**xtended ER (EER) to model inheritance-like structures.
- A **super-entity** has shared attributes.
- **sub-entities** specialize it (e.g., STUDENT, STAFF).
- Subtypes can have specific relationships (e.g. STAFF has CONTRACT)
- Disjoint vs overlapping; total vs partial.
  - Supertype: **PERSON**; subtypes: **STUDENT**, **STAFF** (U-marks = children; 1/2 lines = partial/total coverage).
  - Circle label: **d** = disjoint, **o** = overlapping.



Transforming Abstraction Structures (EER)

There are (at least) four common approaches:

1. **Way 1**: one relation for each super- and sub-entity set; sub-relation key includes super key.
2. **Way 2**: one relation for each sub-entity set; include both sub and super attributes.
3. **Way 3**: one relation for the whole hierarchy; add one `type` attribute (may lead to many NULLs).
4. **Way 4**: like Way 3, but use boolean "flags" (e.g., `isStaff`, `isStudent`).



Way 1:
PERSON(PersonId,Name)
STAFF(PersonId, room)
STUDENT(PersonId,Major)
FK: STAFF.PersonId→PERSON.PersonId,
STUDENT.PersonId→PERSON.PersonId
Way 3:
PERSON(PersonId,Name, Room, Major,
IsStaff:bool, IsStudent:bool)

Heuristics for Abstraction Structures (translated labels)

- **Disjoint + Total**: often choose Way 2 (or Way 3 if acceptable NULLs).
- **Disjoint + Partial**: often choose Way 1 (avoid repetition).
- **Overlapping + Total**: often choose Way 4 (flags), otherwise Way 1.
- **Overlapping + Partial**: often choose Way 4; otherwise, Way 1 (be careful with repetition).

Key Decision Principle: Use keys to enforce constraints and avoid redundancy.

Applying Rules (practical advice)

- Real ER diagrams may trigger multiple rules; there are exceptions.
- A general approach:
  1. transform entity sets,
  2. transform attributes,
  3. transform relationship sets,
  4. transform abstraction structures.
- If several rules could apply:
  - try suitable rules sequentially,
  - compare results from the database viewpoint (fewer relations, clearer structure, fewer NULLs).
- All relationships (except identifying ones) can always be transformed via Rule 8 by creating a relationship relation (but you may need to normalize later).
- Way 1 for abstraction structures always works, but can cause repetition.

Summary

- The relational model represents data as relations (tables) with strict structural properties.
- Keys: candidate keys and primary keys ensure identifiability; foreign keys represent references and enforce integrity.
- Transformation rules provide a systematic mapping from ER/EER designs to relational schemas.
- When multiple transformations are possible, choose the one that fits the domain and leads to a clean schema.