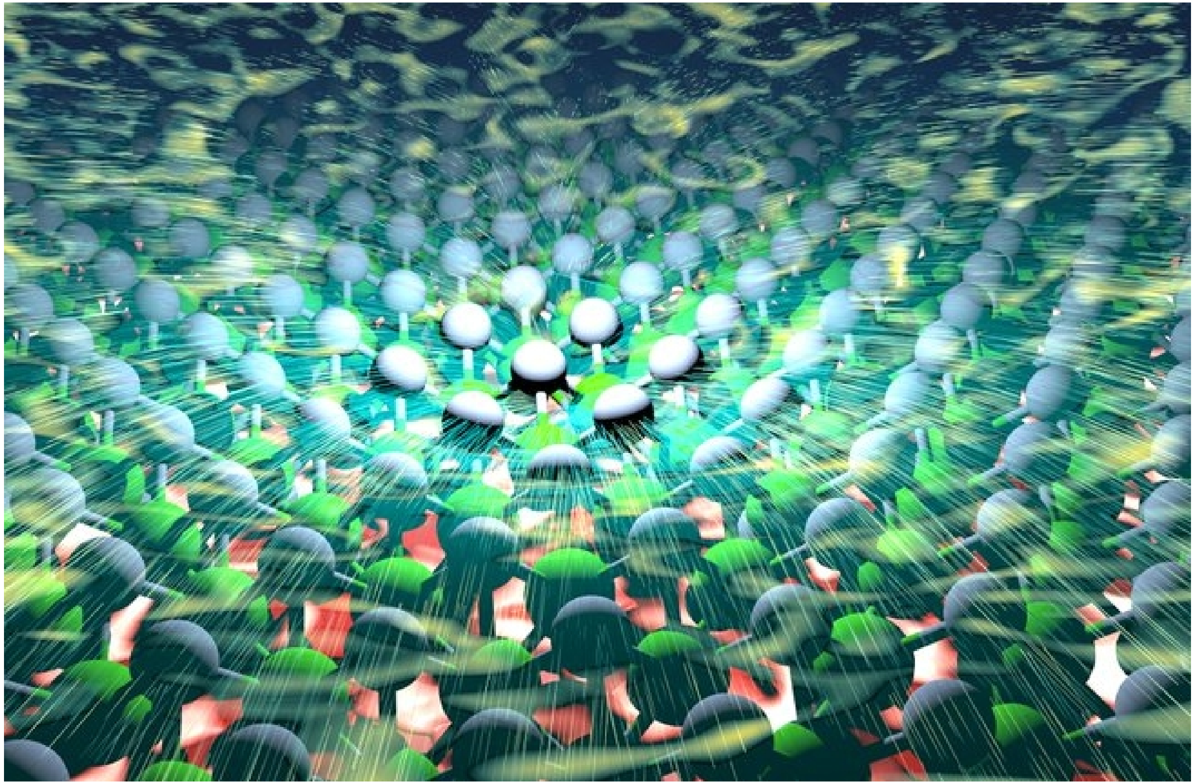
Superconductivity in 2D materials



Jyväskylä Summer School "Emergent Quantum Matter in Artificial Two-dimensional Materials"

Tuesday, August 9th 2022

Schedule for the lecture

- 40 min lecture
- 15 min break
- 40 min lecture
- 15 min break
- 40 min lecture

Today's plan

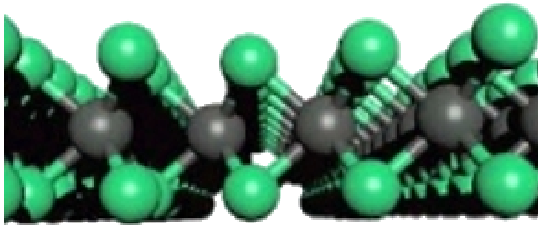
- Van der Waals superconductors
- The impact of superconductivity in an electronic structure
- Conventional and unconventional superconductivity
- Impurities in 2D superconductors
- Topological superconductivity

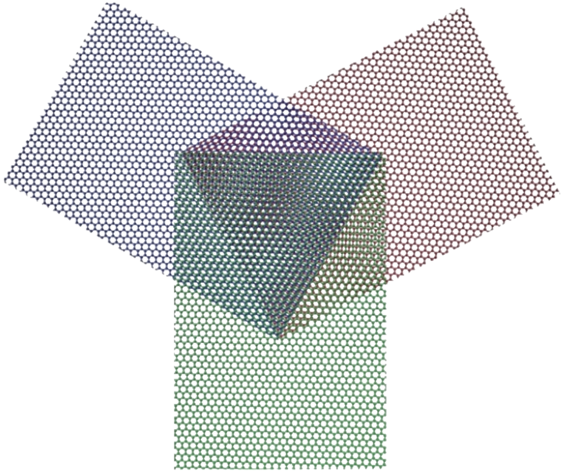
Superconducting van der Waals materials

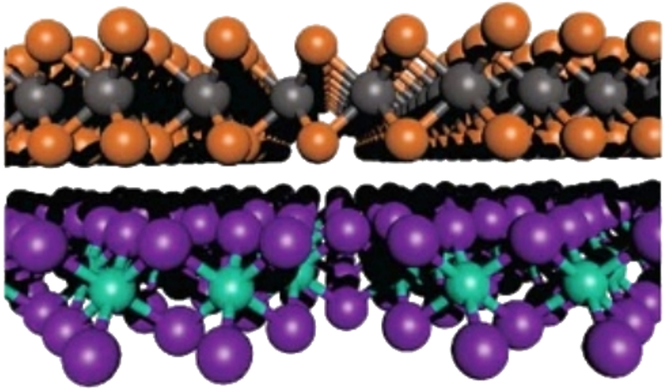
Twisted graphene multilayers

$NbSe_2$

$CrBr_3/NbSe_2$







Gapped trivial superconductors

Nodal superconductivity
Spin-triplet superconductivity

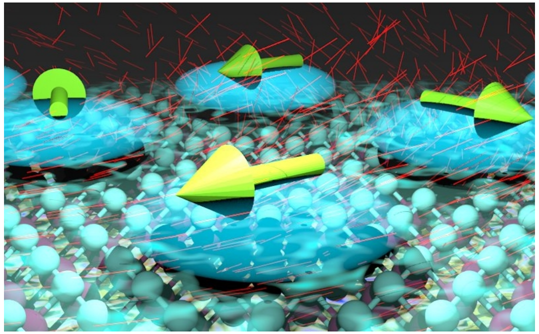
Gapped topological superconductors

The role of electronic interactions

Electronic interactions are responsible for symmetry breaking

Broken time-reversal symmetry

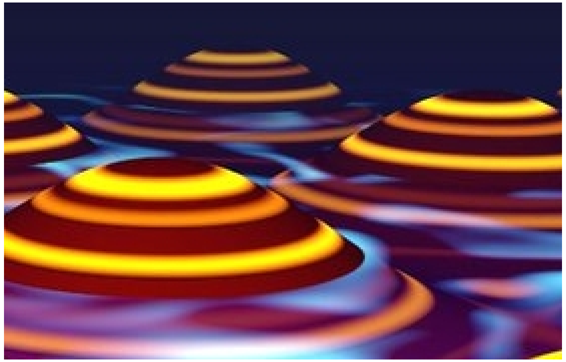
Classical magnets



$$\mathbf{M} \to -\mathbf{M}$$

Broken crystal symmetry

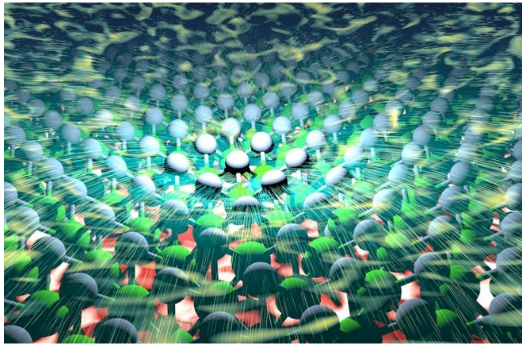
Charge density wave



$$\mathbf{r} \to \mathbf{r} + \mathbf{R}$$

Broken gauge symmetry

Superconductors



$$\langle c_\uparrow c_\downarrow \rangle \to e^{i\phi} \langle c_\uparrow c_\downarrow \rangle$$

Quantum matter with interactions

We can consider two broad groups of interacting quantum matter

$$H = \sum_{ij} t_{ij} c_i^\dagger c_j + \sum_{ijkl} V_{ijkl} c_i^\dagger c_j c_k^\dagger c_l$$

With a mean field description

$$H \approx \sum_{ij} \bar{t}_{ij} c_i^\dagger c_j + \sum_{ij} \Delta_{ij} c_i c_j$$

Approximate quadratic Hamiltonian
Effective single particle description

Weakly correlated matter

Without a mean field description

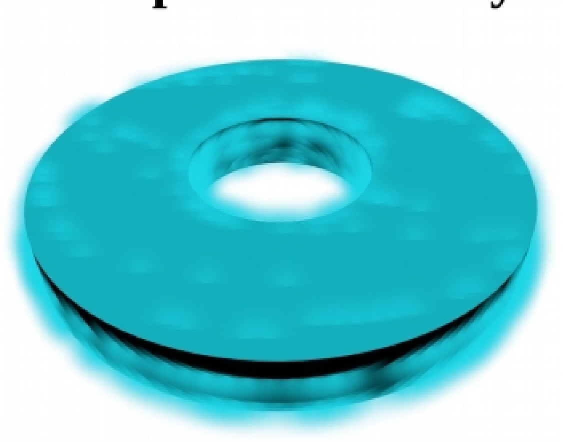
$$????$$

No good quadratic approximation
Requires exact solutions or numerical

Strongly correlated matter

Macroscopic quantum phenomena

Superconductivity

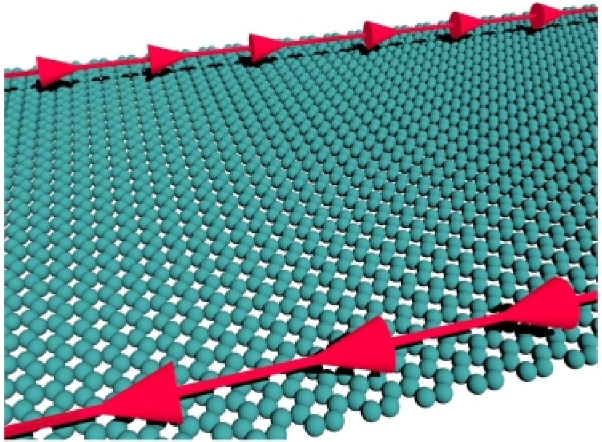


$$\Phi = \frac{h}{2e}$$

Many-body state

<u>Quantization</u> of flux

Quantum Hall effect



$$\sigma_{xy} = \nu \frac{e^2}{h}$$

Single-particle state

<u>Quantization</u> of conductance

The theoretical description of superconductivity

Interactions and mean field

Free Hamiltonian ***Interactions***

$$H = \sum_{ij} t_{ij} c_i^\dagger c_j + \sum_{ijkl} V_{ijkl} c_i^\dagger c_j c_k^\dagger c_l$$

What are these interactions coming from?

- Electrostatic (repulsive) interactions
- Mediated by other quasiparticles (phonons, magnons, plasmons,...)

The net effective interaction can be attractive or repulsive

<u>Superconductivity requires effective attractive interactions</u>

Origin of attractive interactions

Interactions between electrons can be effectively attractive when mediated by other quasiparticles

Conventional superconductors

Phonons

Unconventional superconductors

Antiferromagnetic magnons

Ferromagnetic magnons

Plasmons

Valence fluctuations

Charge fluctuations

...

A simple interacting Hamiltonian

Free Hamiltonian

Interactions (Hubbard term)

$$H = \sum_{ij} t_{ij} [c^\dagger_{i\uparrow} c_{j\uparrow} + c^\dagger_{i\downarrow} c_{j\downarrow}] + \sum_i U c^\dagger_{i\uparrow} c_{i\uparrow} c^\dagger_{i\downarrow} c_{i\downarrow}$$

From now on lets consider we have a spin degree of freedom $\uparrow, \downarrow$

What is the ground state of this Hamiltonian?

$U < 0$ Superconductivity

$U > 0$ Magnetism

The mean-field approximation, superconductivity

Mean field: *Approximate four fermions by two fermions times expectation values*

Four fermions (not exactly solvable)

Two fermions (exactly solvable)

$$U c^\dagger_{i\uparrow} c_{i\uparrow} c^\dagger_{i\downarrow} c_{i\downarrow} \approx U \langle c^\dagger_{i\uparrow} c^\dagger_{i\downarrow} \rangle c_{i\uparrow} c_{i\downarrow} + h.c.$$

$$U c^\dagger_{i\uparrow} c_{i\uparrow} c^\dagger_{i\downarrow} c_{i\downarrow} \approx \Delta c_{i\uparrow} c_{i\downarrow} + h.c.$$

For $U < 0$

i.e. attractive interactions

$\Delta \sim \langle c^\dagger_{i\uparrow} c^\dagger_{i\downarrow} \rangle$ is the superconducting order

A Hamiltonian for a superconductor

Free Hamiltonian **Pairing term**

$$H = \sum_{ij} t_{ij} [c^\dagger_{i\uparrow} c_{j\uparrow} + c^\dagger_{i\downarrow} c_{j\downarrow}] + \Delta \sum_i c_{i\uparrow} c_{i\downarrow} + h.c.$$

Lets have a look to the term $\Delta c_{i\uparrow} c_{i\downarrow}$

Superconductivity and symmetries

$$H \sim \Delta c_{i\uparrow} c_{i\downarrow} + h.c.$$

$$H|GS\rangle = E_{GS}|GS\rangle$$

This term destroys two electrons

$|GS\rangle$ Ground state

The ground state can not have a well defined number of electrons

$$|GS\rangle \sim |2e\rangle + |4e\rangle + |6e\rangle + ...$$

$$|GS\rangle \sim |1e\rangle + |3e\rangle + |5e\rangle + ...$$

Gauge symmetry and superconductivity

What we know from quantum mechanics

“The phase of a wavefunction (field operator) does not have physical meaning”

This is what we know as gauge symmetry

$$c_n \to e^{i\phi} c_n$$
$$c_n^\dagger \to e^{-i\phi} c_n^\dagger$$

Terms in the Hamiltonian that do not change under this transformation

$c_n^\dagger c_m$	$c_{n,\uparrow}^\dagger c_{n,\uparrow} - c_{n,\downarrow}^\dagger c_{n,\downarrow}$	$c_{n,\uparrow}^\dagger c_{m,\downarrow}$	$c_{n,\uparrow}^\dagger c_{m,\uparrow} c_{n,\downarrow}^\dagger c_{m,\downarrow}$
Hopping	*Magnetism*	*Spin-orbit coupling*	*Interactions*

Superconductivity and gauge symmetry breaking

Gauge symmetry

$$c_n \to e^{i\phi} c_n \qquad\qquad c_n^\dagger \to e^{-i\phi} c_n^\dagger$$

How does the superconducting order transform under a gauge transformation?

$$\Delta = \langle c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger \rangle$$

Superconductivity and gauge symmetry breaking

Gauge symmetry

$$c_n \rightarrow e^{i\phi} c_n \qquad c_n^\dagger \rightarrow e^{-i\phi} c_n^\dagger$$

How does the superfluid density transform under a gauge transformation?

$$\Delta \rightarrow e^{-2i\phi} \Delta$$

A superconductor breaks gauge symmetry

The electronic structure of a superconductor

The Nambu representation

How do we solve a Hamiltonian of the form $H = \sum_{\mathbf{k},s} \epsilon_{\mathbf{k}} c^\dagger_{\mathbf{k},s} c_{\mathbf{k},s} + \sum_{\mathbf{k}} \Delta c^\dagger_{\mathbf{k},\uparrow} c^\dagger_{-\mathbf{k},\downarrow} + h.c.$

Define a Nambu spinor $$\Psi_{\mathbf{k}} = \begin{pmatrix} c_{\mathbf{k}\uparrow} \\ c_{\mathbf{k}\downarrow} \\ c^\dagger_{-\mathbf{k}\downarrow} \\ -c^\dagger_{-\mathbf{k}\uparrow} \end{pmatrix}$$

Electron sector

Hole sector

The Hamiltonian in the Nambu basis is quadratic and can be diagonalized

$$H = \Psi^\dagger_{\mathbf{k}} \mathcal{H} \Psi_{\mathbf{k}}$$

A single orbital superconductor

The original Hamiltonian $$H = \sum_{\mathbf{k},s} \epsilon_{\mathbf{k}} c^\dagger_{\mathbf{k},s} c_{\mathbf{k},s} + \sum_{\mathbf{k}} \Delta c^\dagger_{\mathbf{k},\uparrow} c^\dagger_{-\mathbf{k},\downarrow} + h.c.$$

Can be rewritten as

$$H = \frac{1}{2} \Psi^\dagger_{\mathbf{k}} \mathcal{H} \Psi_{\mathbf{k}} \qquad \Psi_{\mathbf{k}} = \begin{pmatrix} c_{\mathbf{k}\uparrow} \\ c_{\mathbf{k}\downarrow} \\ c^\dagger_{-\mathbf{k}\downarrow} \\ -c^\dagger_{-\mathbf{k}\uparrow} \end{pmatrix}$$

Electron sector

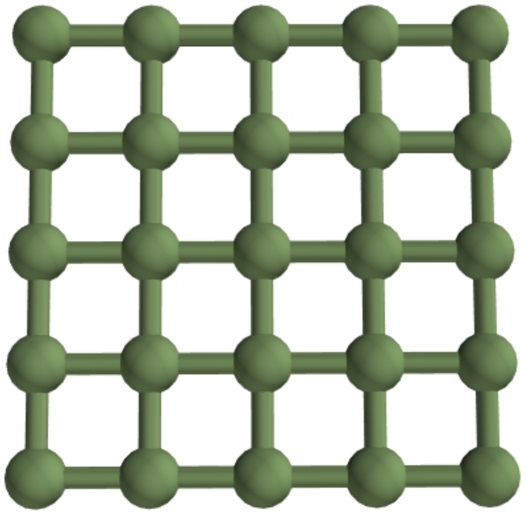
Hole sector

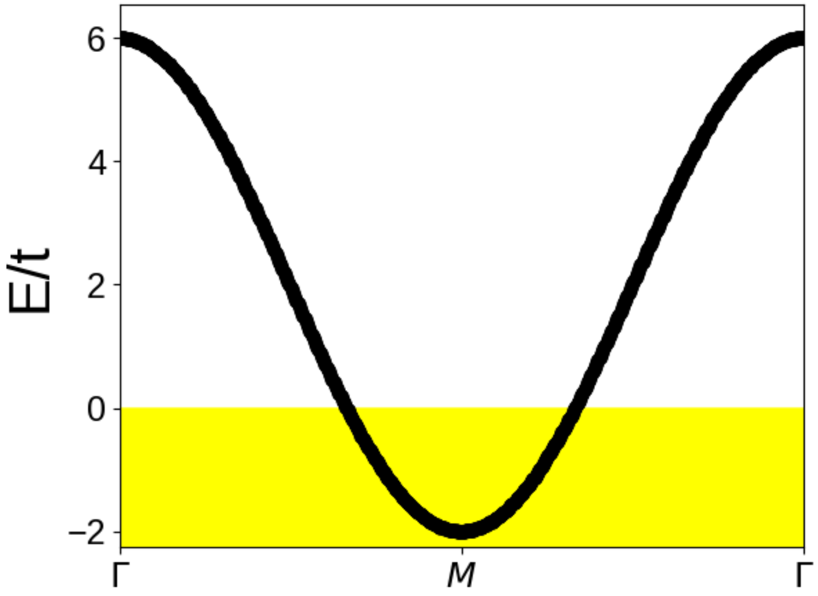
with $$\mathcal{H} = \begin{pmatrix} \epsilon_{\mathbf{k}} & 0 & \Delta & 0 \\ 0 & \epsilon_{\mathbf{k}} & 0 & \Delta \\ \Delta & 0 & -\epsilon_{\mathbf{k}} & 0 \\ 0 & \Delta & 0 & -\epsilon_{\mathbf{k}} \end{pmatrix}$$

A single orbital superconductor

Single orbital in the square lattice

$$H = \sum_{\mathbf{k},s} \epsilon_{\mathbf{k}} c^\dagger_{\mathbf{k},s} c_{\mathbf{k},s}$$

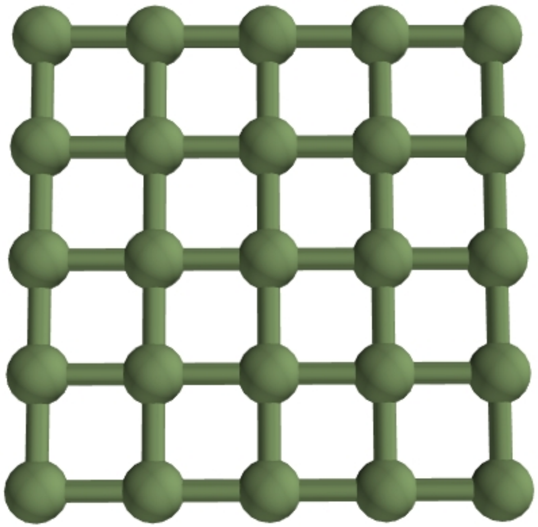




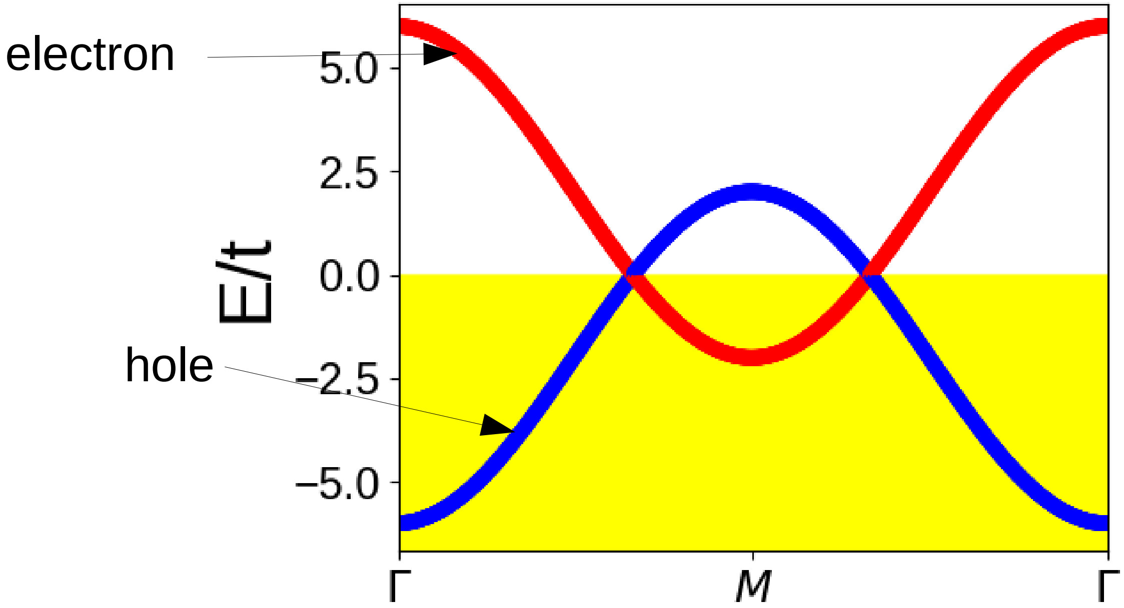
A single orbital superconductor

Single orbital in the square lattice

$$H = \sum_{\mathbf{k},s} \epsilon_{\mathbf{k}} c^\dagger_{\mathbf{k},s} c_{\mathbf{k},s} + \sum_{\mathbf{k}} \Delta c^\dagger_{\mathbf{k},\uparrow} c^\dagger_{-\mathbf{k},\downarrow} + h.c.$$



$$\Delta \to 0$$



A single orbital superconductor

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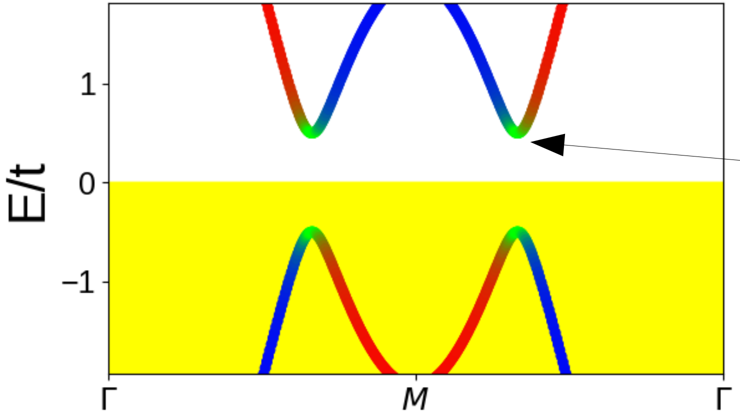
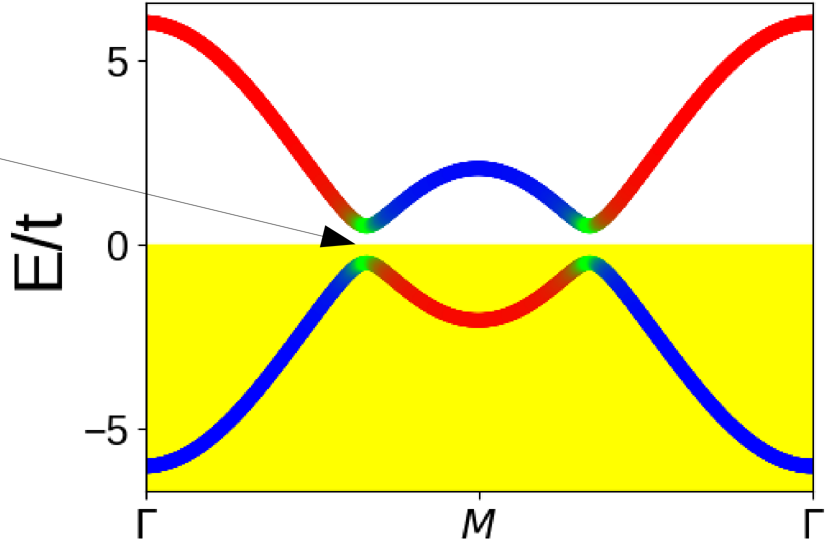
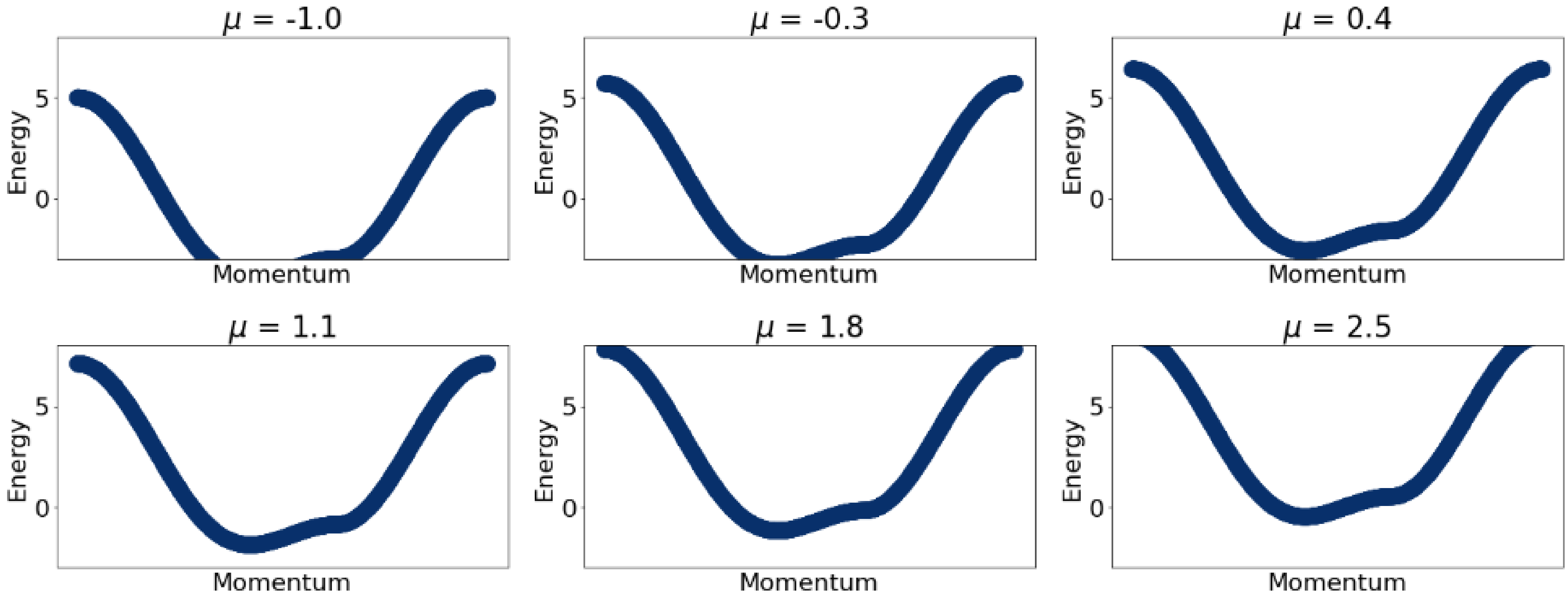


SC gap



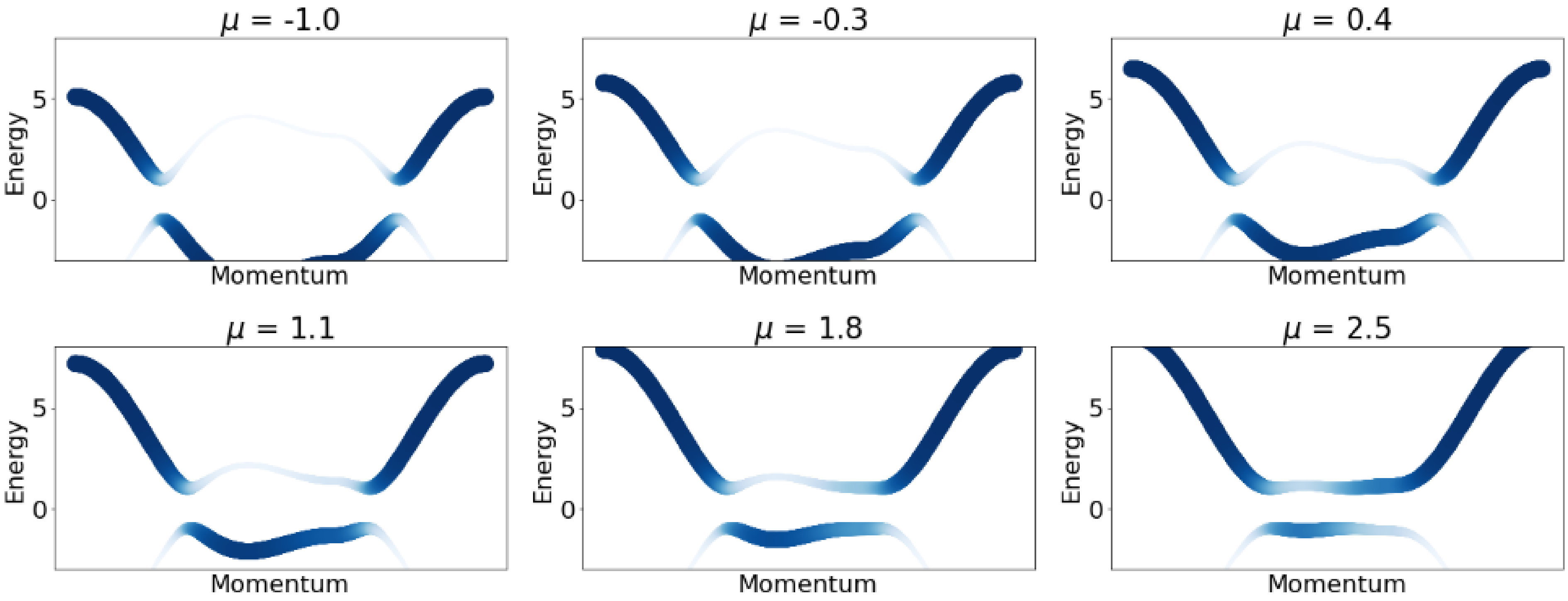
Impact of superconductivity in the electronic structure

Let us take the electronic structure of a triangular lattice



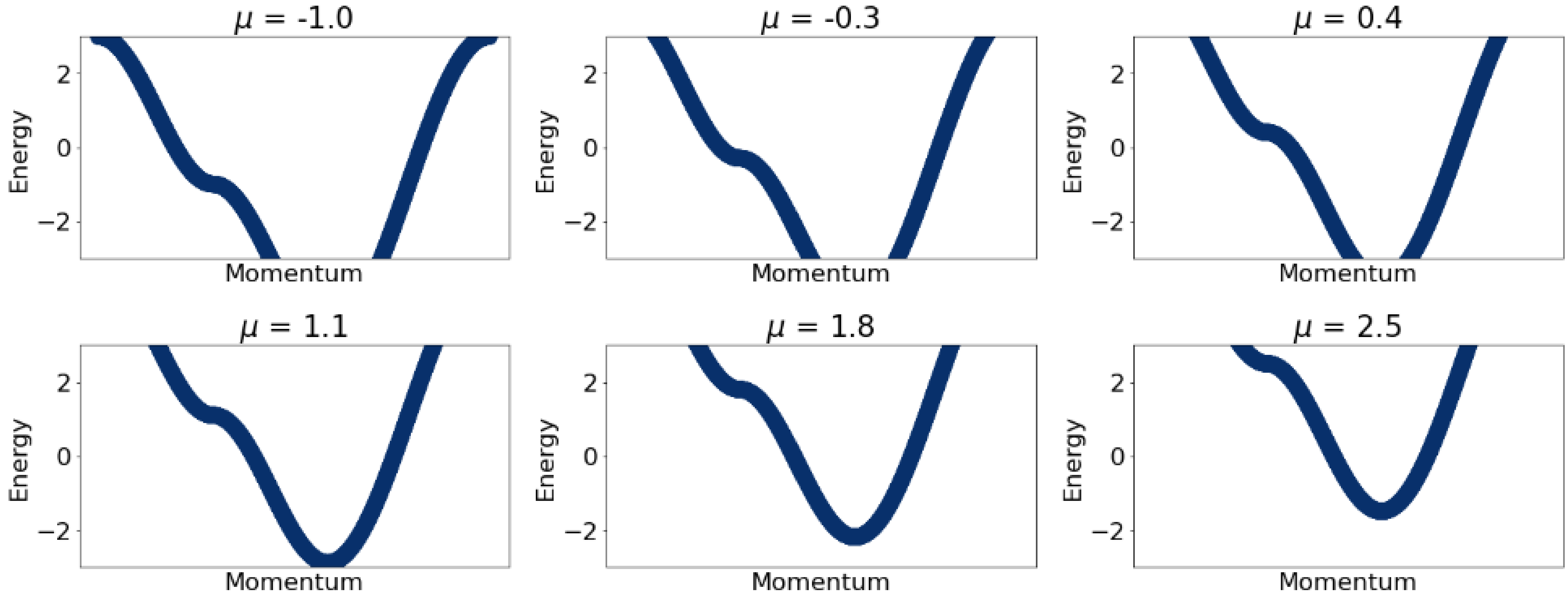
Impact of superconductivity in the electronic structure

A uniform superconducting terms opens up a gap in the electronic structure



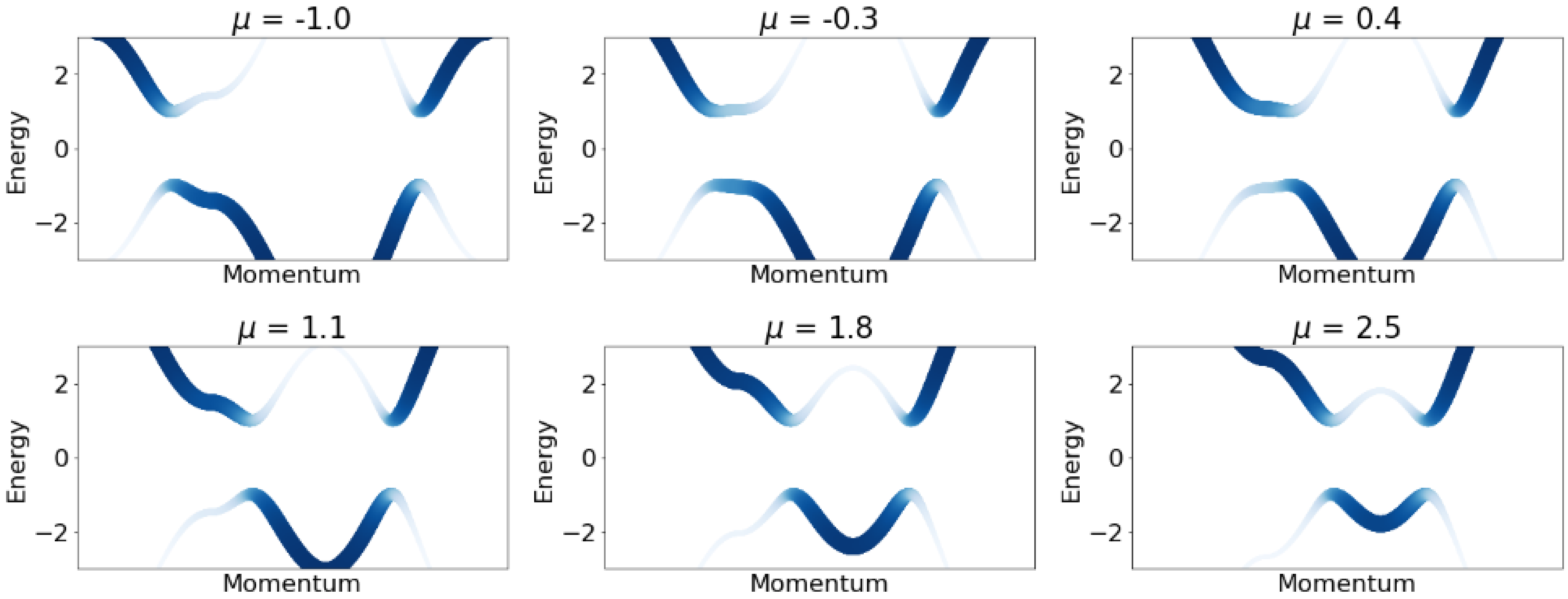
Impact of superconductivity in the electronic structure

Let us take the electronic structure of a square lattice



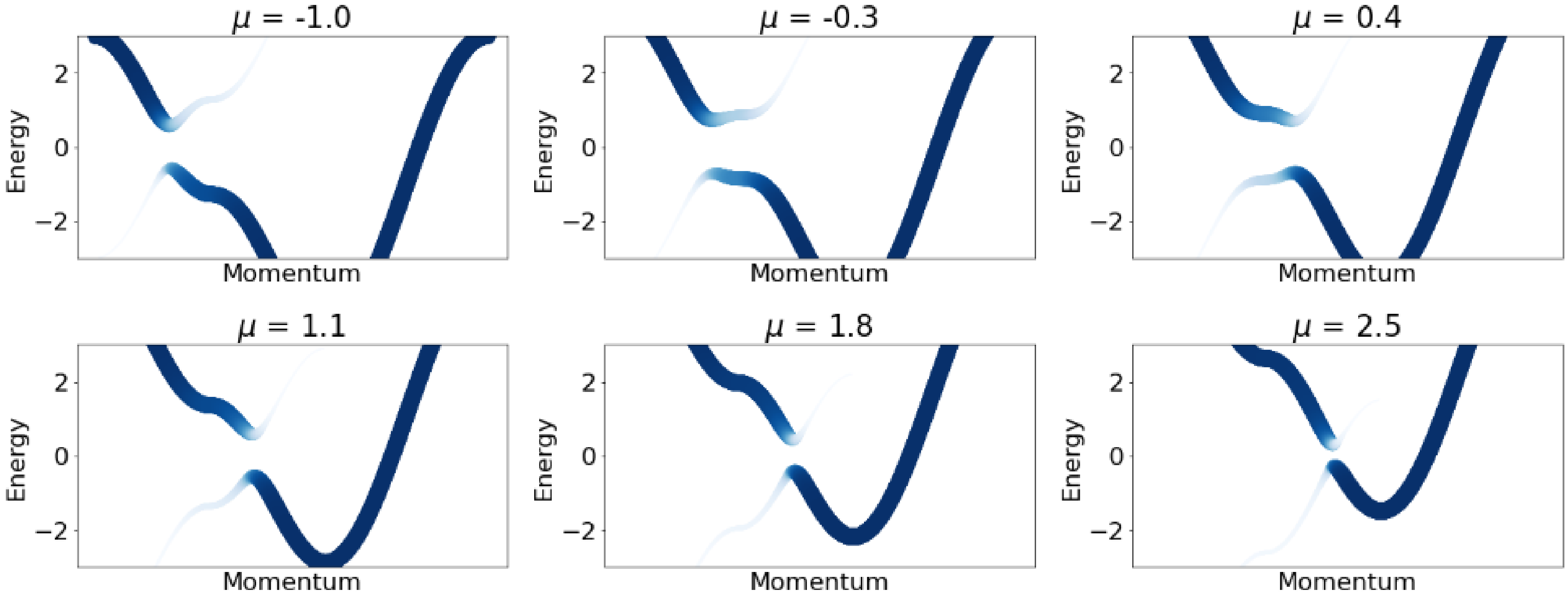
Impact of superconductivity in the electronic structure

A uniform superconducting terms opens up a gap in the electronic structure



Nodal superconductivity

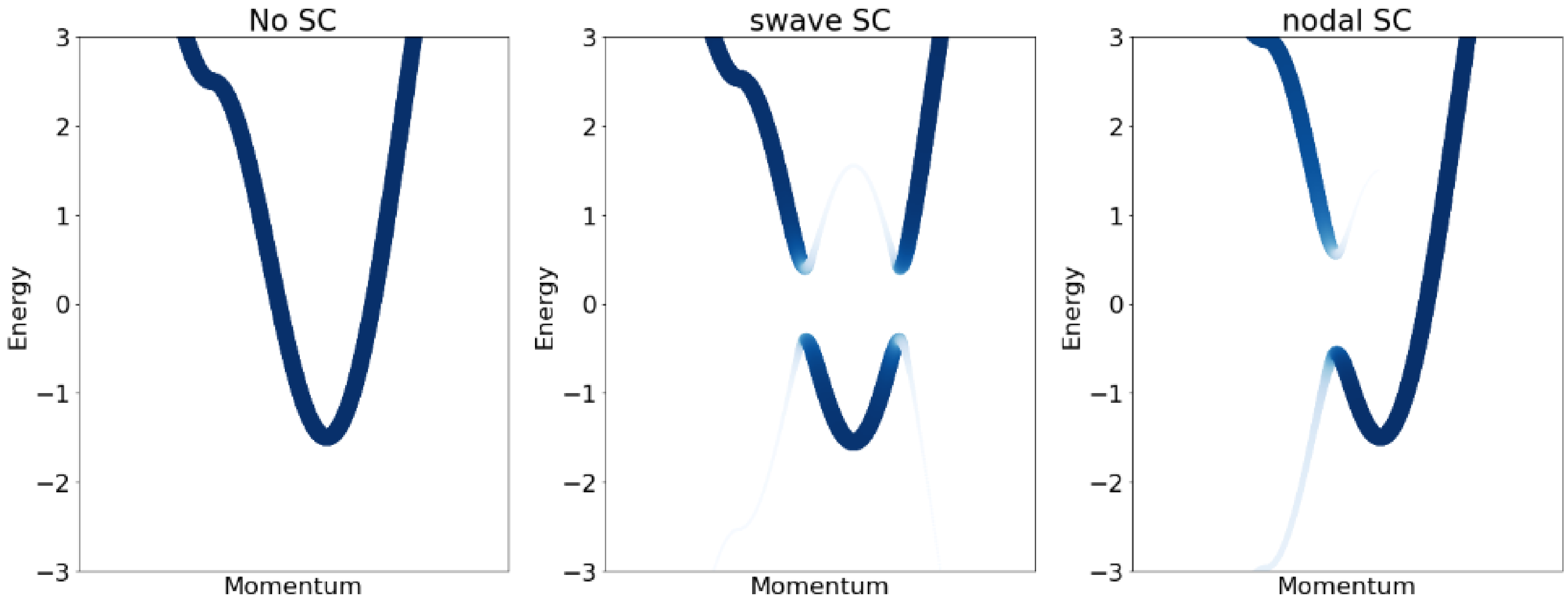
Let us now take a square lattice, and add a nodal superconducting order parameter



Some parts of the Fermi surface get a gap, while others remain gapless

Gapped and gapless superconductivity

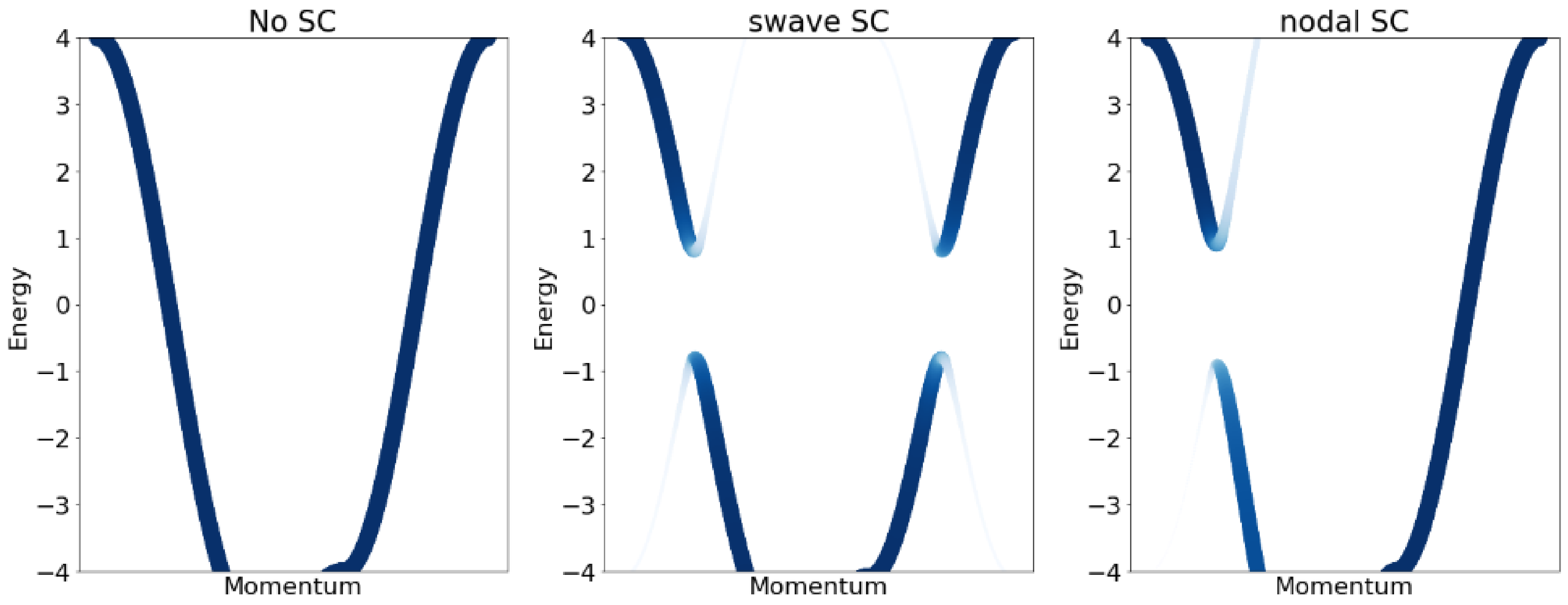
The electronic structure is modified differently depending on the type of superconductivity



Square lattice

Gapped and gapless superconductivity

The electronic structure is modified differently depending on the type of superconductivity



Triangular lattice

Break

10-15 min break

(optional) to discuss during the break

What is the correct sign for this equality, and why?

$$\langle c^\dagger_{k,\uparrow} c^\dagger_{-k,\uparrow} \rangle = \langle c^\dagger_{-k,\uparrow} c^\dagger_{k,\uparrow} \rangle$$

$$\langle c^\dagger_{k,\uparrow} c^\dagger_{-k,\uparrow} \rangle = -\langle c^\dagger_{-k,\uparrow} c^\dagger_{k,\uparrow} \rangle$$

Unconventional superconductivity

Generic forms of superconductivity

A generic superconducting Hamiltonian

$$\widehat{H}' = \sum_{\boldsymbol{k},\sigma} \epsilon_{\boldsymbol{k}} \, \widehat{c}^{\dagger}_{\boldsymbol{k},\sigma} \, \widehat{c}_{\boldsymbol{k},\sigma} - \frac{1}{2} \sum_{\boldsymbol{k}}{}' \sum_{\sigma_1,\sigma_2} \left[\Delta_{\boldsymbol{k},\sigma_1\sigma_2} \, \widehat{c}^{\dagger}_{\boldsymbol{k},\sigma_1} \, \widehat{c}^{\dagger}_{-\boldsymbol{k},\sigma_2} + \Delta^{*}_{\boldsymbol{k},\sigma_1\sigma_2} \, \widehat{c}_{\boldsymbol{k},\sigma_1} \, \widehat{c}_{-\boldsymbol{k},\sigma_2} \right]$$

Can be characterized by a superconducting matrix

$$\Delta_{\boldsymbol{k}} = \begin{pmatrix} \Delta_{\boldsymbol{k},\uparrow\uparrow} & \Delta_{\boldsymbol{k},\uparrow\downarrow} \\ \Delta_{\boldsymbol{k},\downarrow\uparrow} & \Delta_{\boldsymbol{k},\downarrow\downarrow} \end{pmatrix}$$

The symmetry of the SC order determines the nature of the SC order

Superconducting momentum symmetries

A generic type of a superconductor is characterized by the order parameter

Real space

$$\Delta_{\uparrow\downarrow}(\mathbf{r},\mathbf{r}') \sim \langle c_{\mathbf{r}\uparrow} c_{\mathbf{r}'\downarrow} \rangle$$

Reciprocal space

$$\Delta_{\uparrow\downarrow}(\mathbf{k}) \sim \langle c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow} \rangle$$

The superconducting state can be characterized by the symmetry of $\Delta_{\uparrow\downarrow}(\mathbf{k})$

Singlet and triplet superconductors

The superconducting order inherits a symmetry property

$$\Delta_{\vec{k},s_1 s_2} = -\Delta_{-\vec{k},s_2 s_1} = \begin{cases} \Delta_{-\vec{k},s_1 s_2} = -\Delta_{\vec{k},s_2 s_1} & \text{even} \\ -\Delta_{-\vec{k},s_1 s_2} = \Delta_{\vec{k},s_2 s_1} & \text{odd} \end{cases}$$

Spin-singlet (even)

$$\Delta_{\uparrow\downarrow}(\mathbf{k}) = \Delta_{\uparrow\downarrow}(-\mathbf{k})$$

Spin-triplet (odd)

$$\Delta_{\uparrow\uparrow}(\mathbf{k}) = -\Delta_{\uparrow\uparrow}(-\mathbf{k})$$

The symmetry of the superconducting order characterizes the superconductor

Generating a spin-triplet superconductor

Let us take a Hamiltonian breaking time-reversal with attractive interactions

$$H = t \sum_{\langle ij \rangle} c^\dagger_{i,s} c_{j,s} + J_z \sum_i \sigma_z^{s,s'} c^\dagger_{i,s} c_{j,s'} + V_1 \sum_{\langle ij \rangle} \sum_s (c^\dagger_{i,s} c^\dagger_{i,s}) \sum_{s'} (c^\dagger_{j,s'} c^\dagger_{j,s'})$$

kinetic *exchange* *attractive interactions*

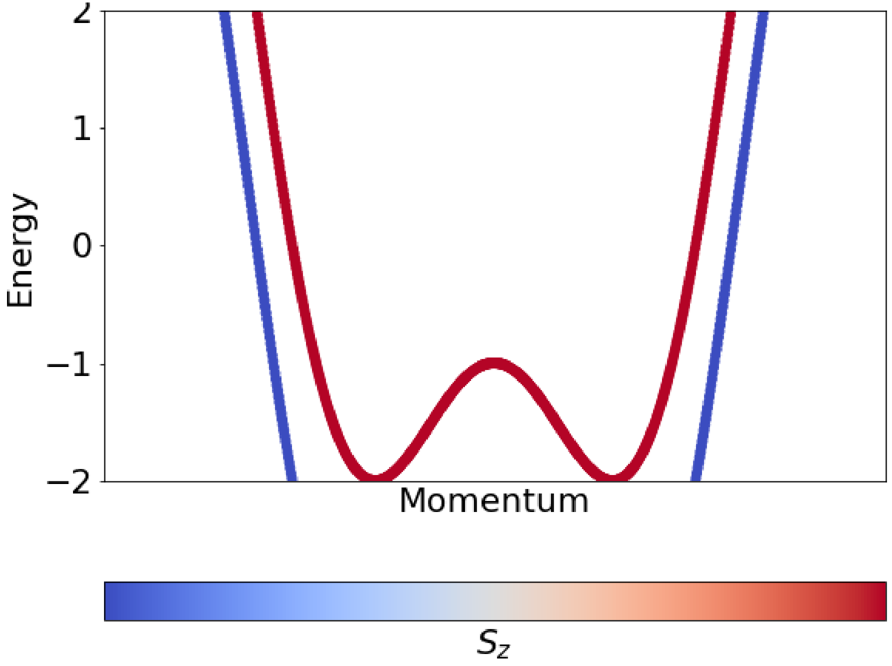
At the mean-field level, this may generate

$$H \sim \sum_{\langle ij \rangle} \Delta_{\uparrow\uparrow} c^\dagger_{i,\downarrow} c^\dagger_{j,\downarrow} + h.c. + ...$$

Generating a spin-triplet superconductor

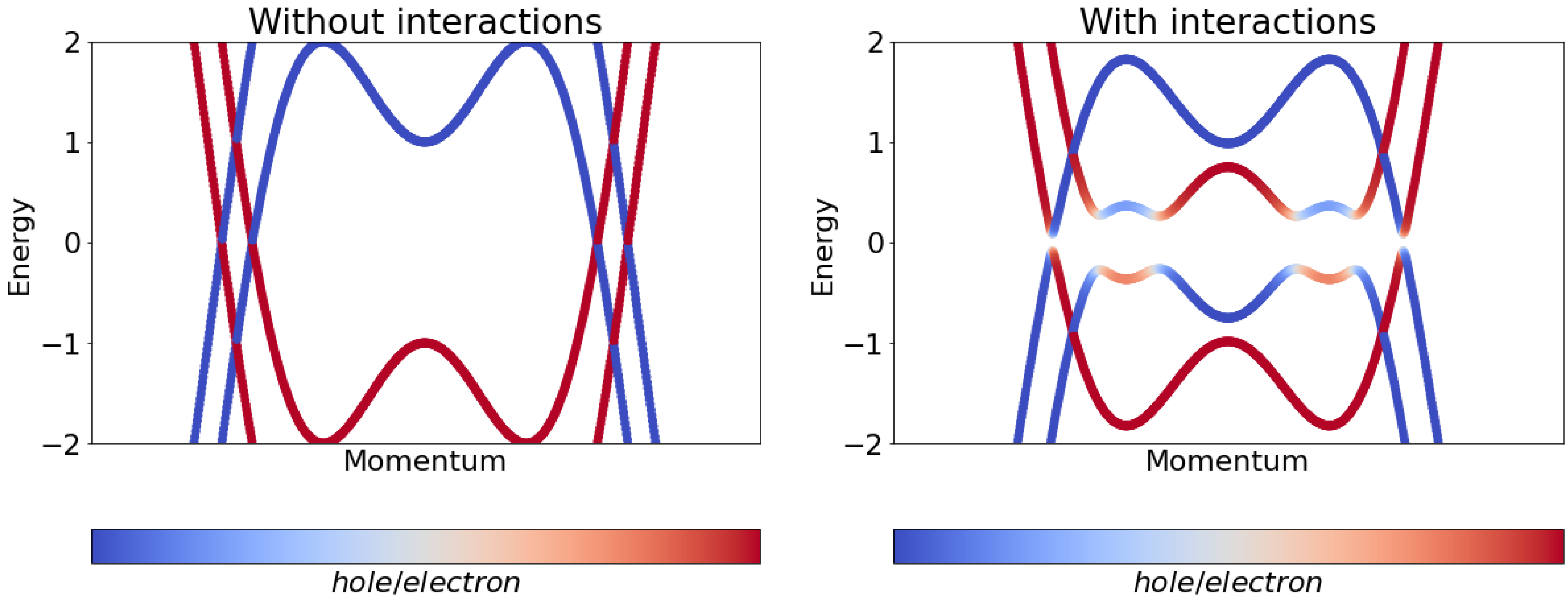
We start with a ferromagnetic 2D material, and see if interactions create superconductivity

$$H = t \sum_{\langle ij \rangle} c^\dagger_{i,s} c_{j,s} + J_z \sum_i \sigma_z^{s,s'} c^\dagger_{i,s} c_{j,s'}$$

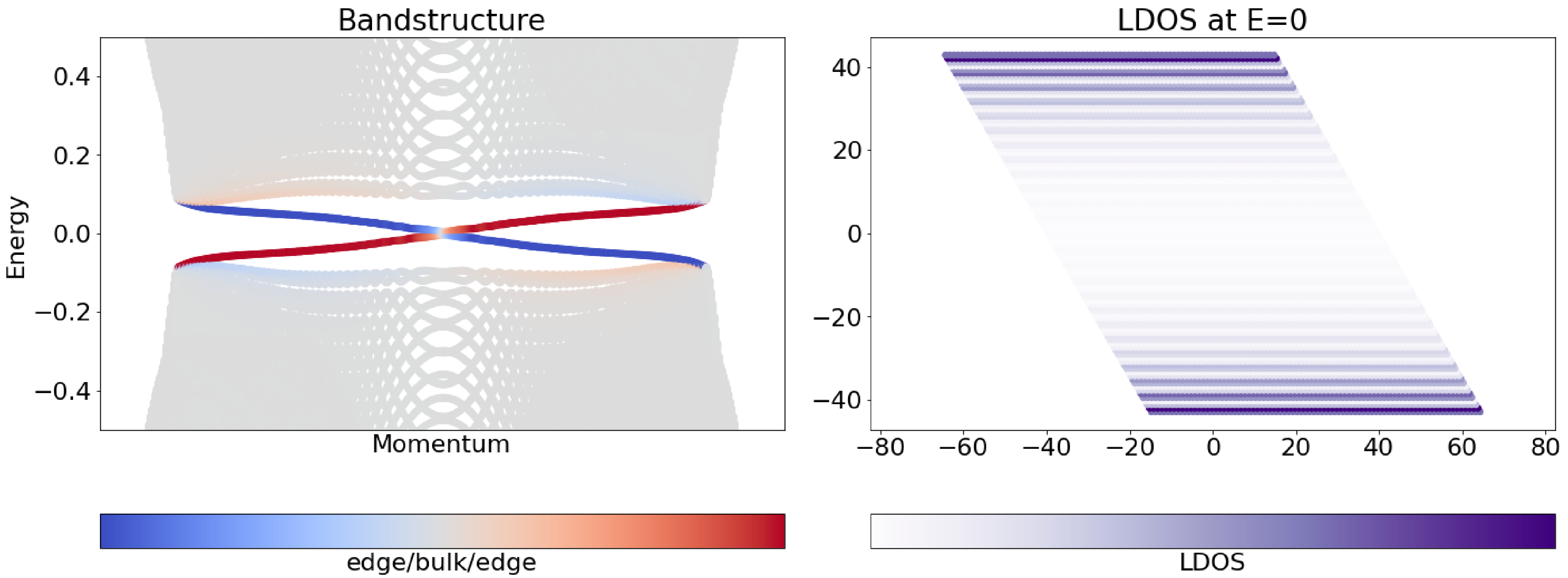


A spin-triplet superconductor

$$H = t\sum_{\langle ij \rangle} c^\dagger_{i,s} c_{j,s} + J_z \sum_i \sigma_z^{s,s'} c^\dagger_{i,s} c_{j,s'} + V_1 \sum_{\langle ij \rangle} \sum_s (c^\dagger_{i,s} c^\dagger_{i,s}) \sum_{s'} (c^\dagger_{j,s'} c^\dagger_{j,s'})$$



A spin-triplet superconductor, in a strip



A ferromagnetic superconductor can develop edge modes

Gapped and gapless superconductors

Let us focus on the superconducting order $\Delta_{\uparrow\downarrow}(\mathbf{k})$

Fully gapped

$$|\Delta_{\uparrow\downarrow}(\mathbf{k})|^2 > 0$$

NbSe$_2$

Gapless

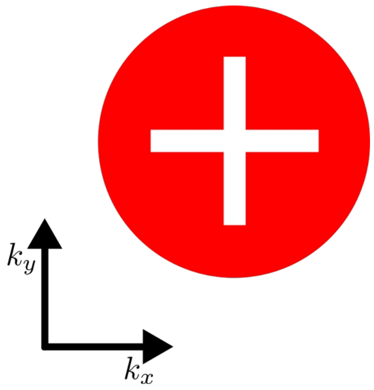
$$|\Delta_{\uparrow\downarrow}(\mathbf{k}_\alpha)|^2 = 0$$

Twisted trilayer graphene

Superconducting momentum symmetries

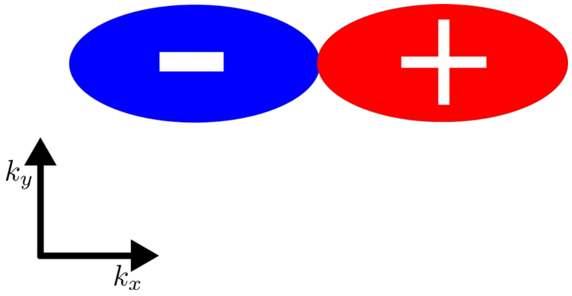
The superconducting state can be characterized by the symmetry of $\Delta(\mathbf{k})$

s-wave



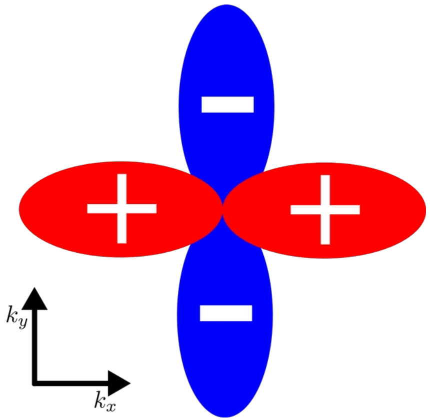
Conventional SC
(driven by phonons)

p-wave



Unconventional SC
(driven by FE magnons)

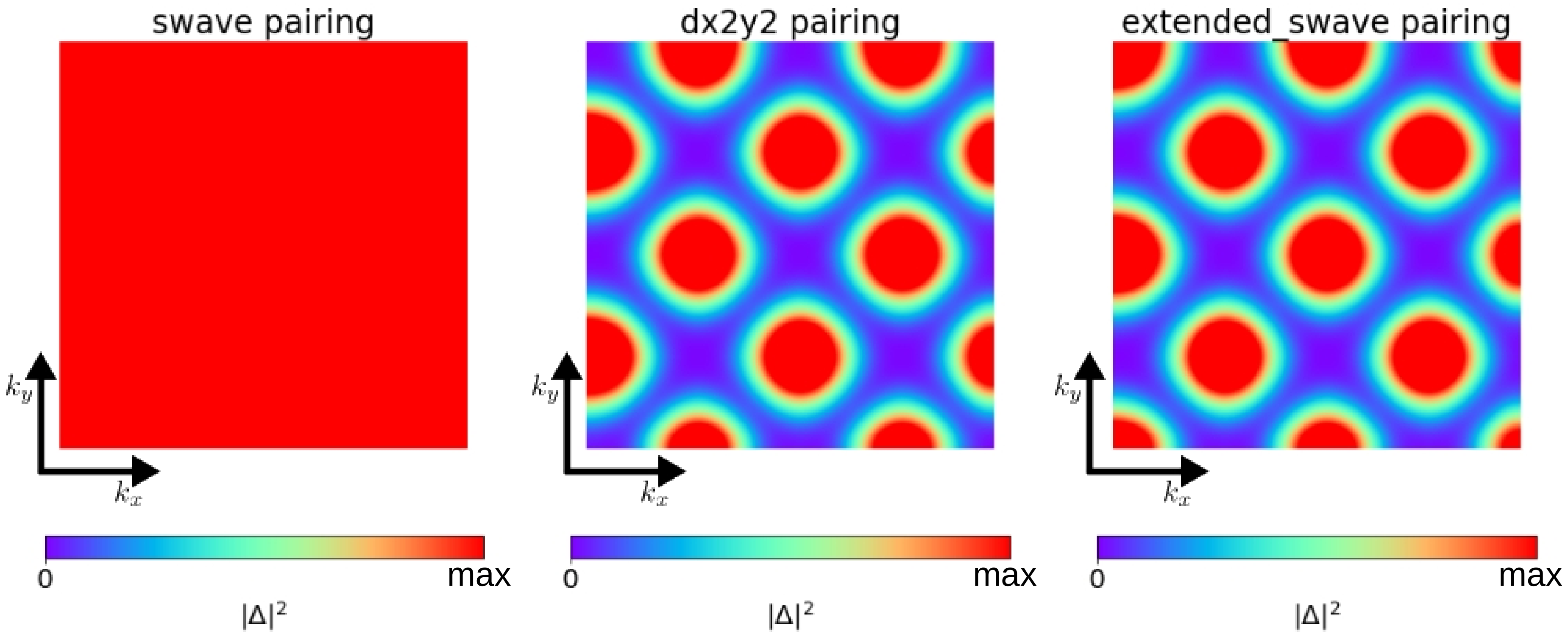
d-wave



Unconventional SC
(driven by AF magnons)

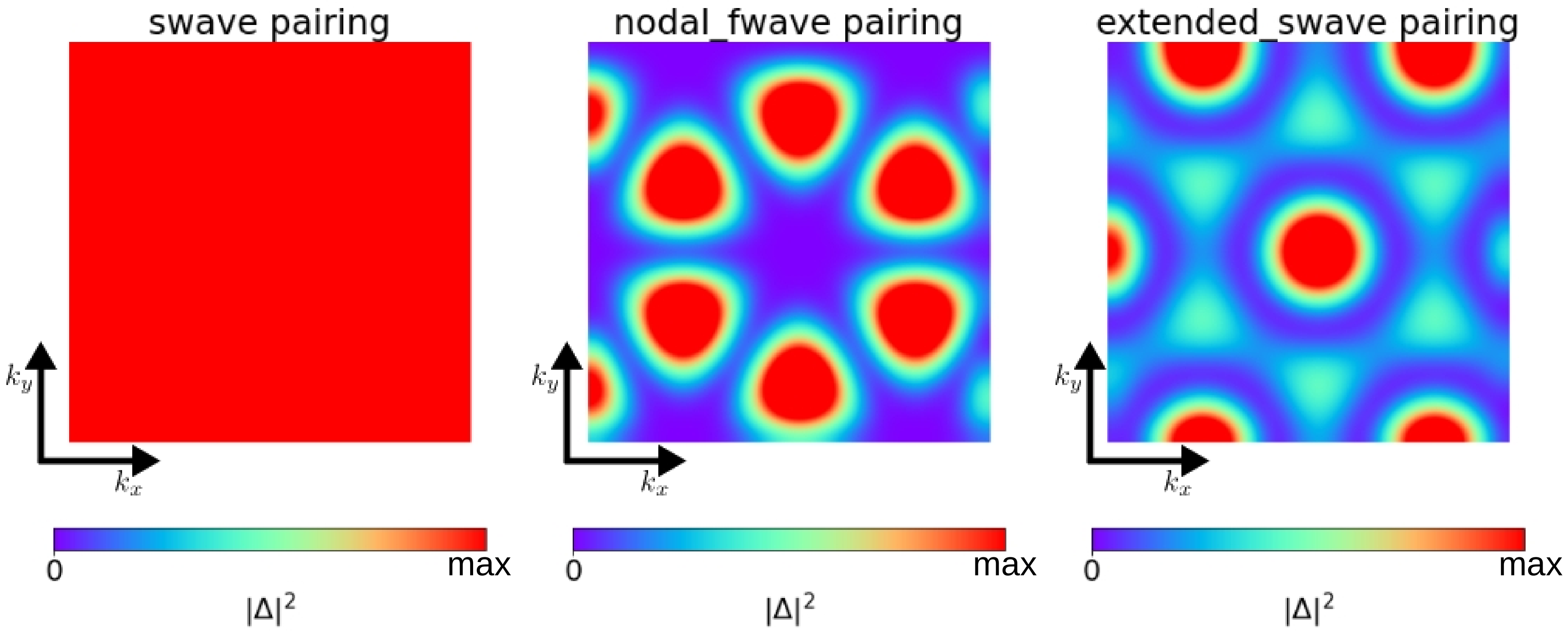
Some superconducting symmetries in the square lattice

Some superconducting symmetries in the square lattice

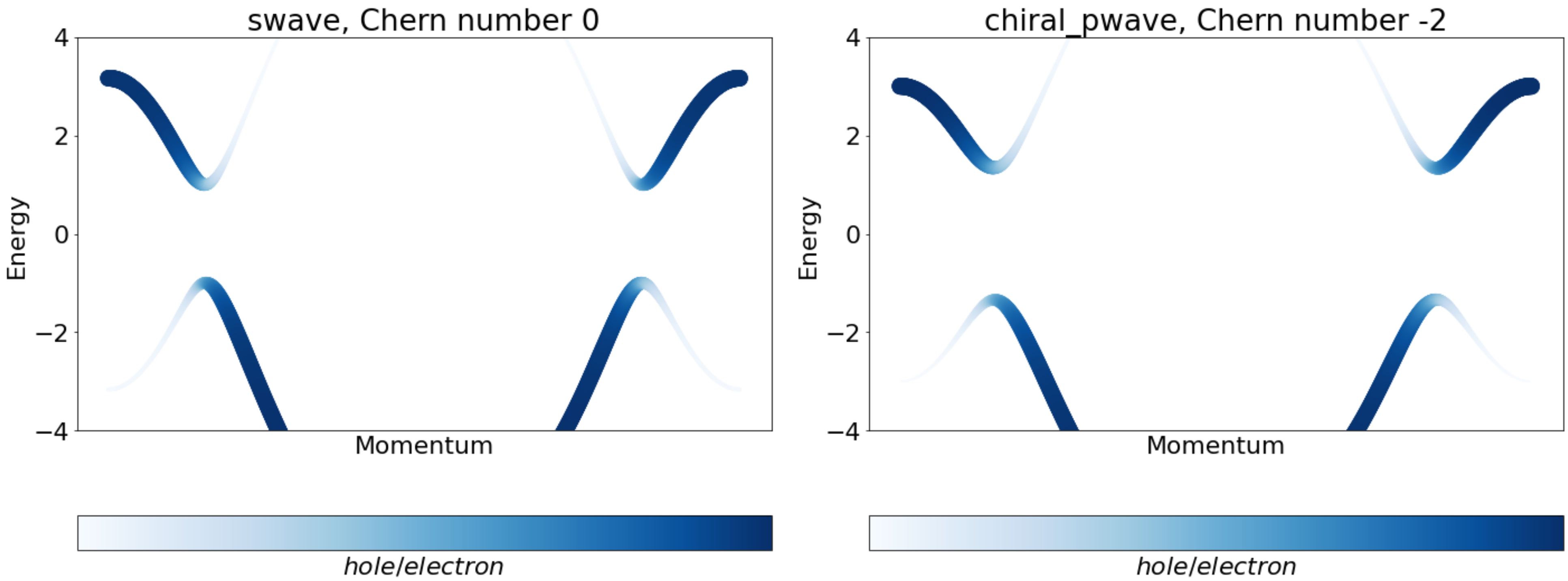


Some superconducting symmetries in the triangular lattice

Some superconducting symmetries in the triangular lattice

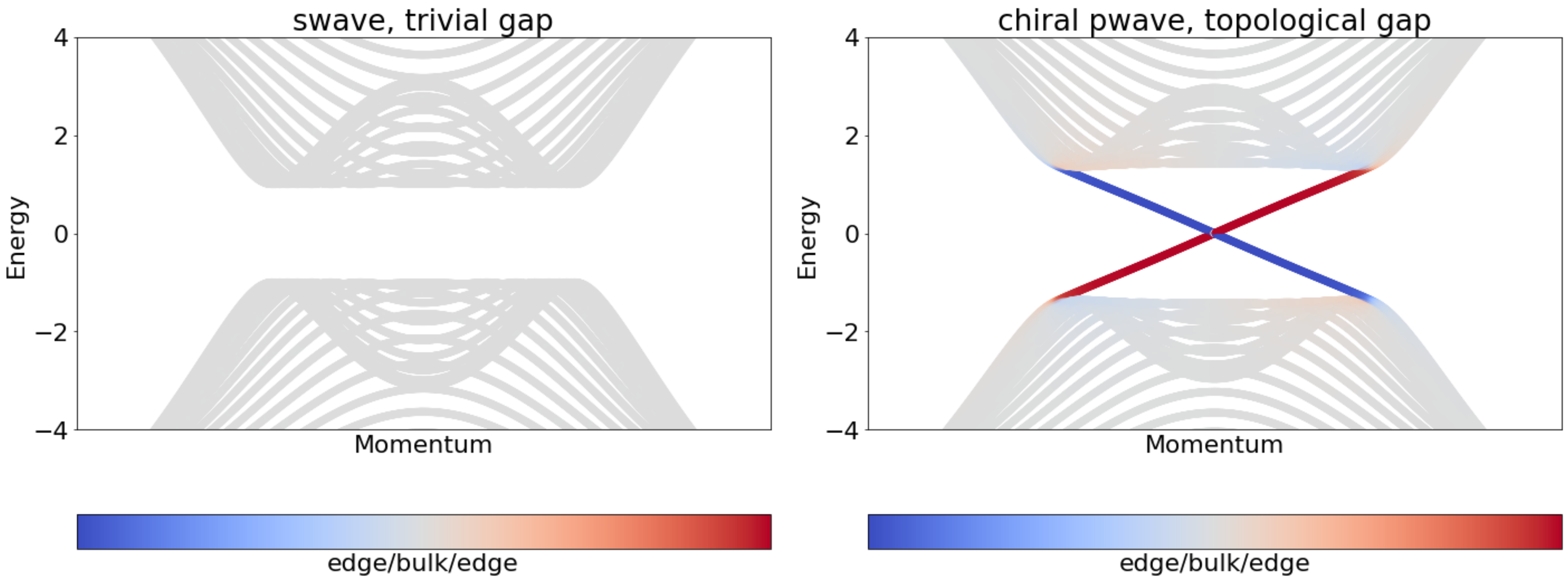


Gapped superconducting orders can be radically different



While both orders are gapped, they have different topological properties

Gapped superconducting orders can be radically different

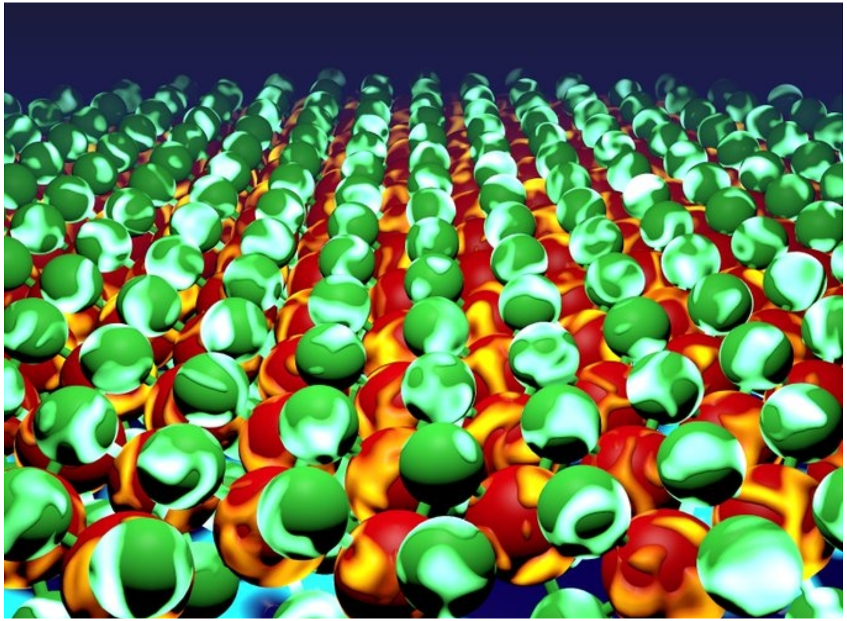


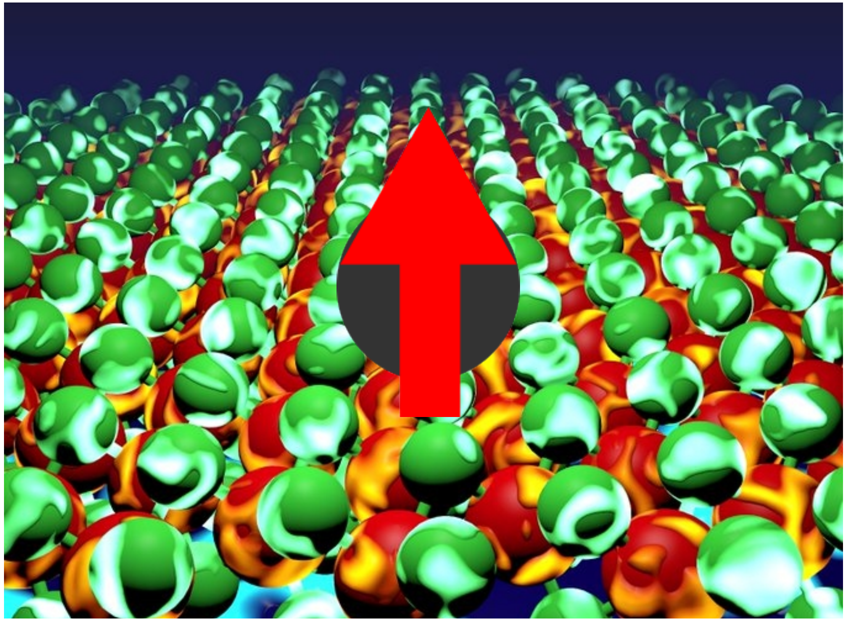
The topological superconducting gap leads to protected edge excitations

Pair breaking effects in superconductors

Impurities in 2D superconductors

So-far we considered pristine superconductors, but what happens when we put impurities?

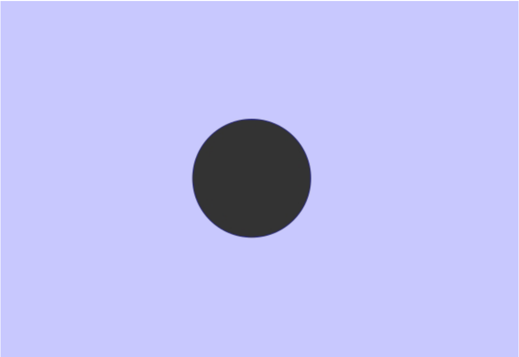




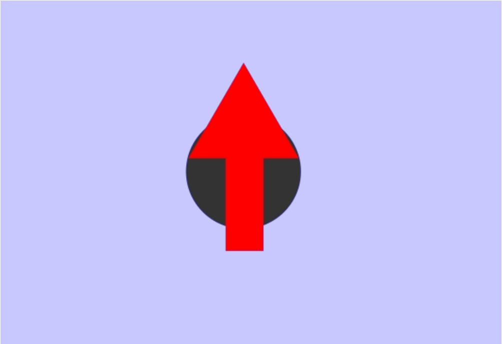
How detrimental are defects in superconductors?

Impurities in 2D superconductors

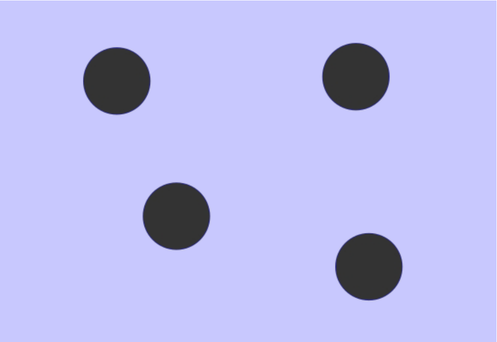
A non-magnetic impurity



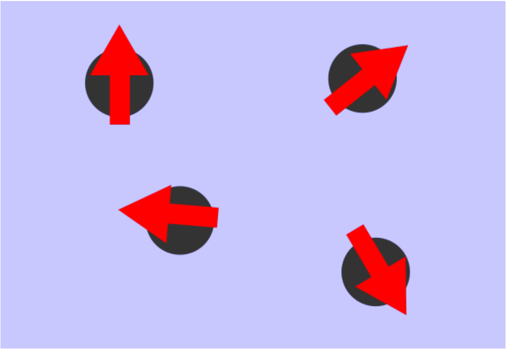
A magnetic impurity



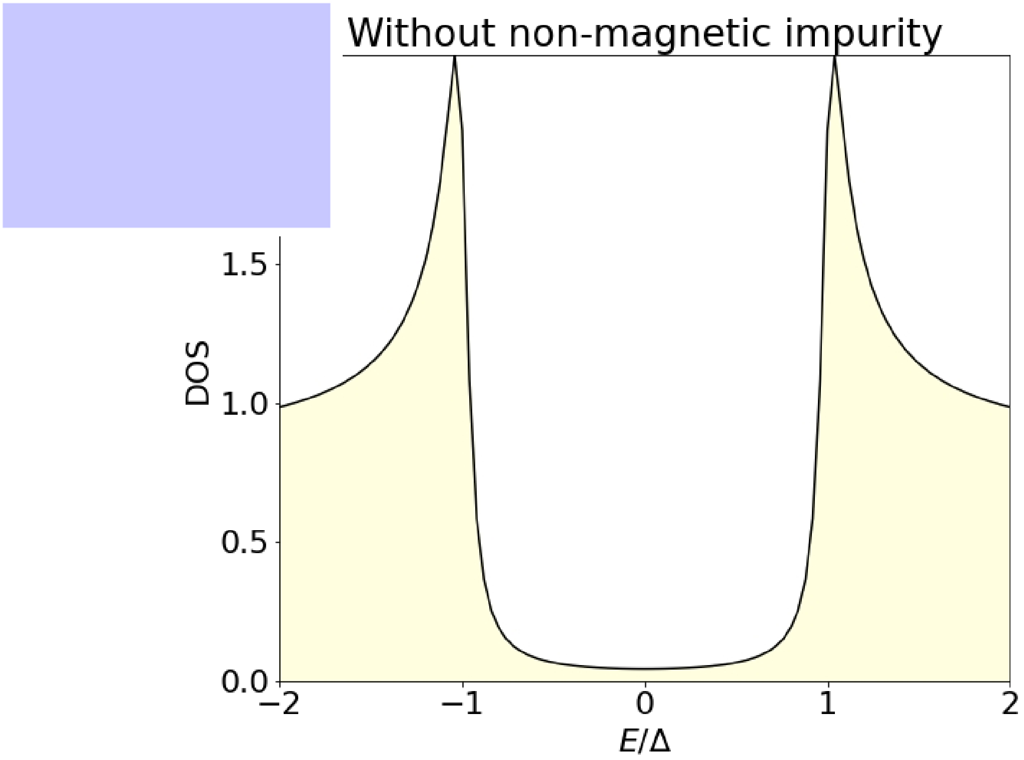
Several non-magnetic impurities

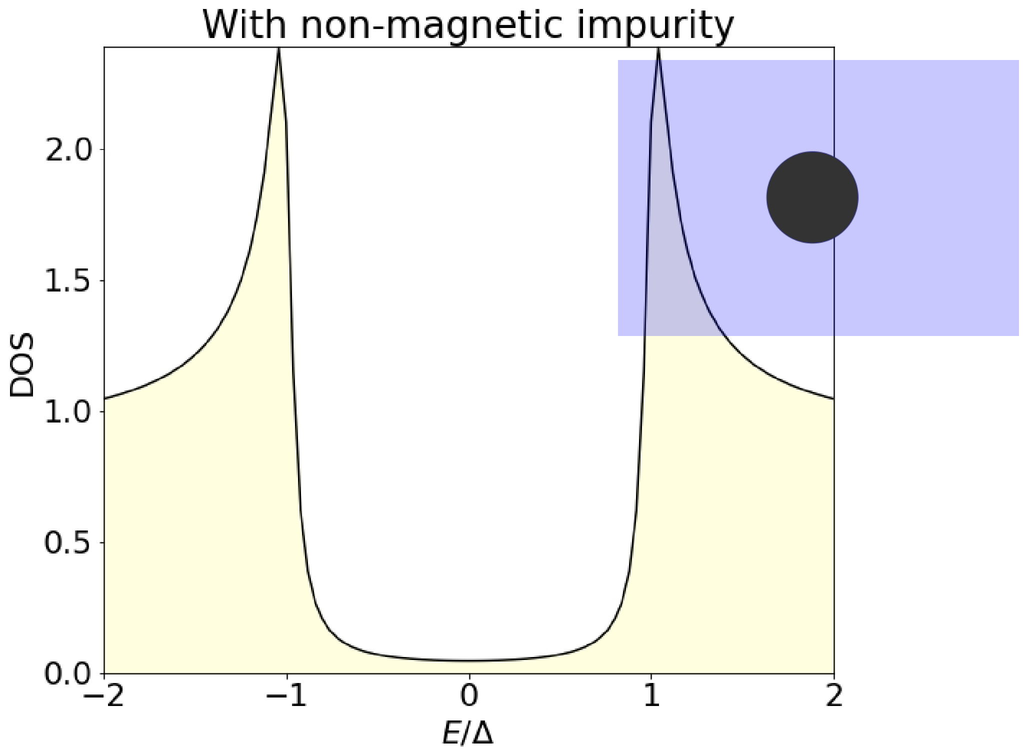


Several magnetic impurities



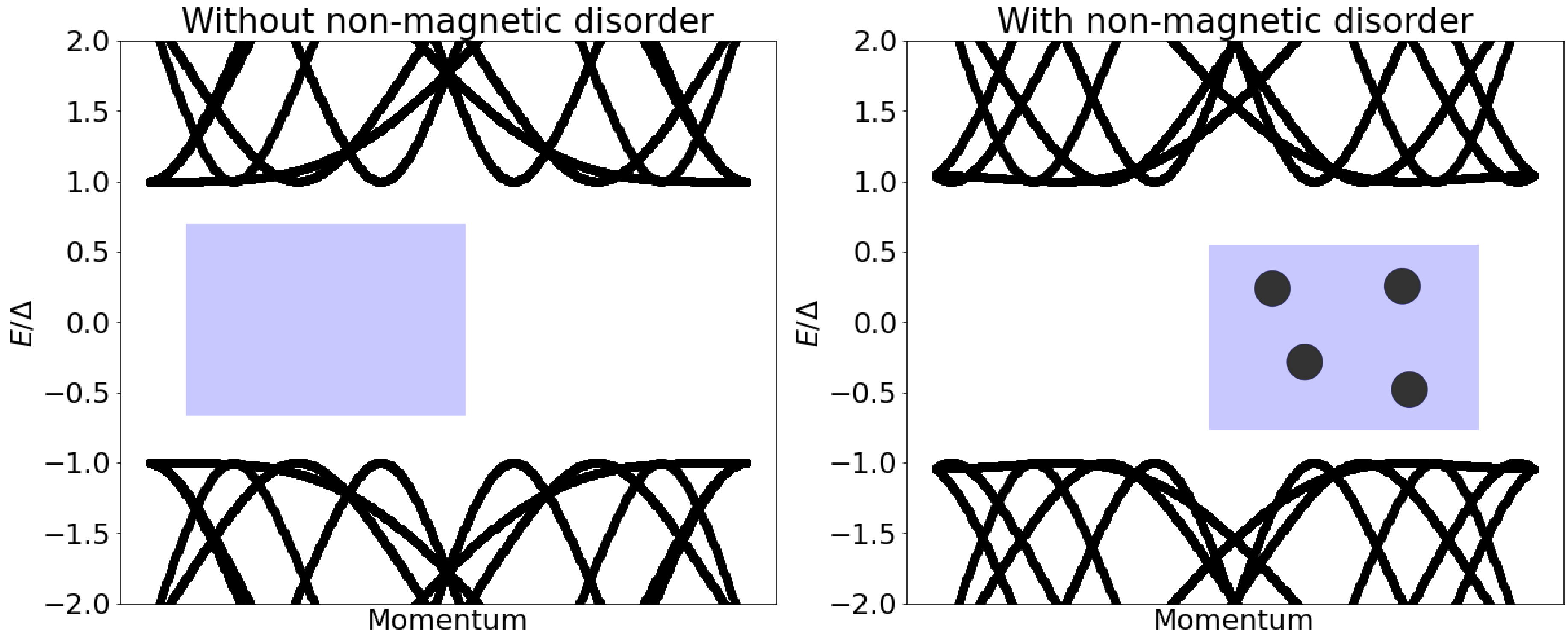
Non-magnetic impurity, conventional s-wave superconductor





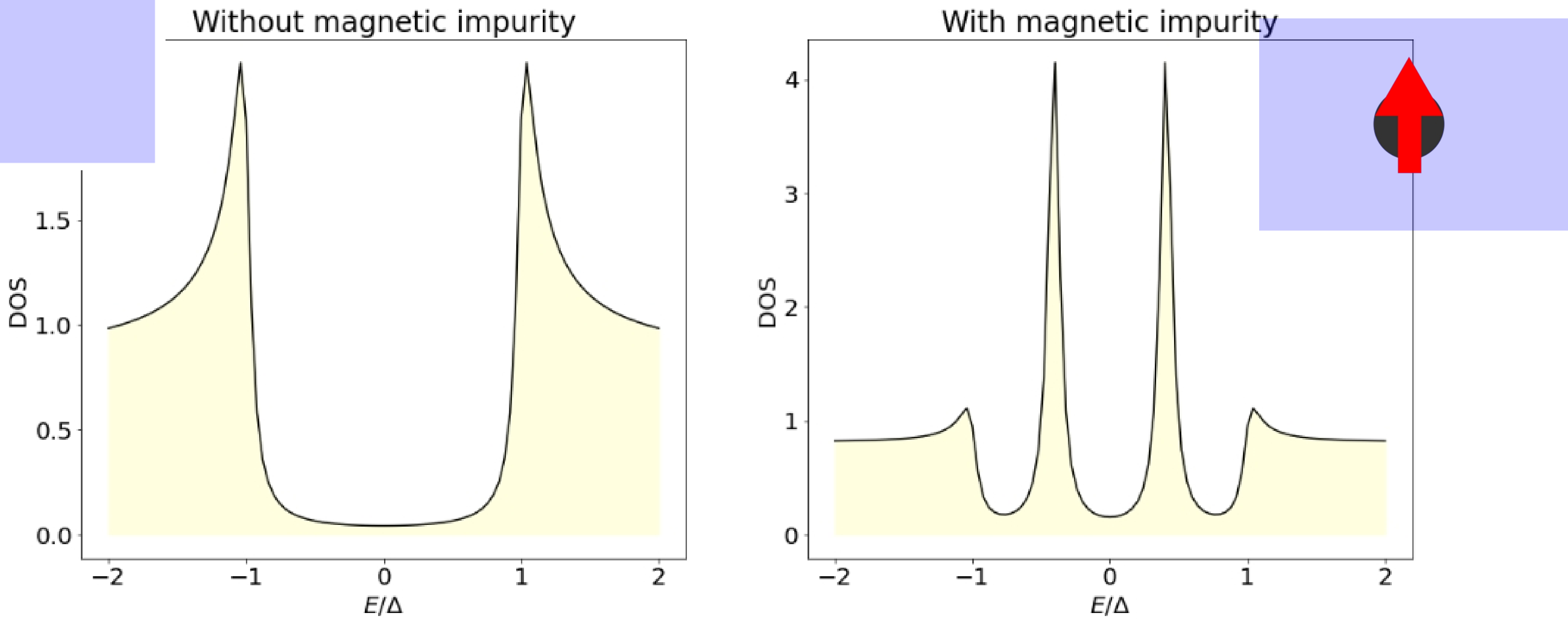
A non-magnetic impurity does not affect conventional s-wave superconductors

Non-magnetic disorder, conventional s-wave superconductor



Non-magnetic disorder does not impact a conventional s-wave superconducting gap

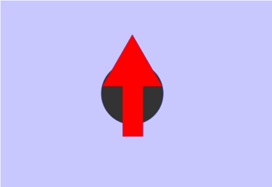
The interplay between magnetism and superconductivity

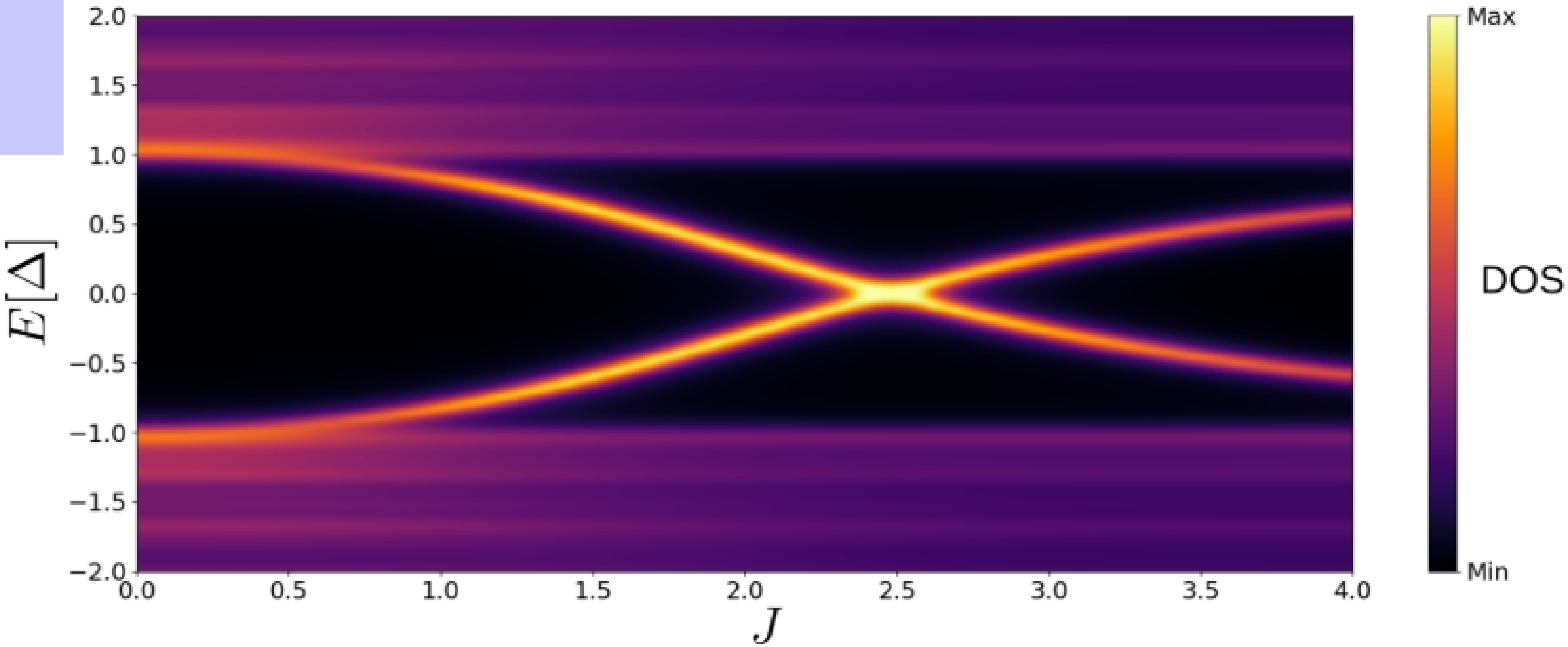


Magnetic impurities create in-gap states in fully gapped superconductors

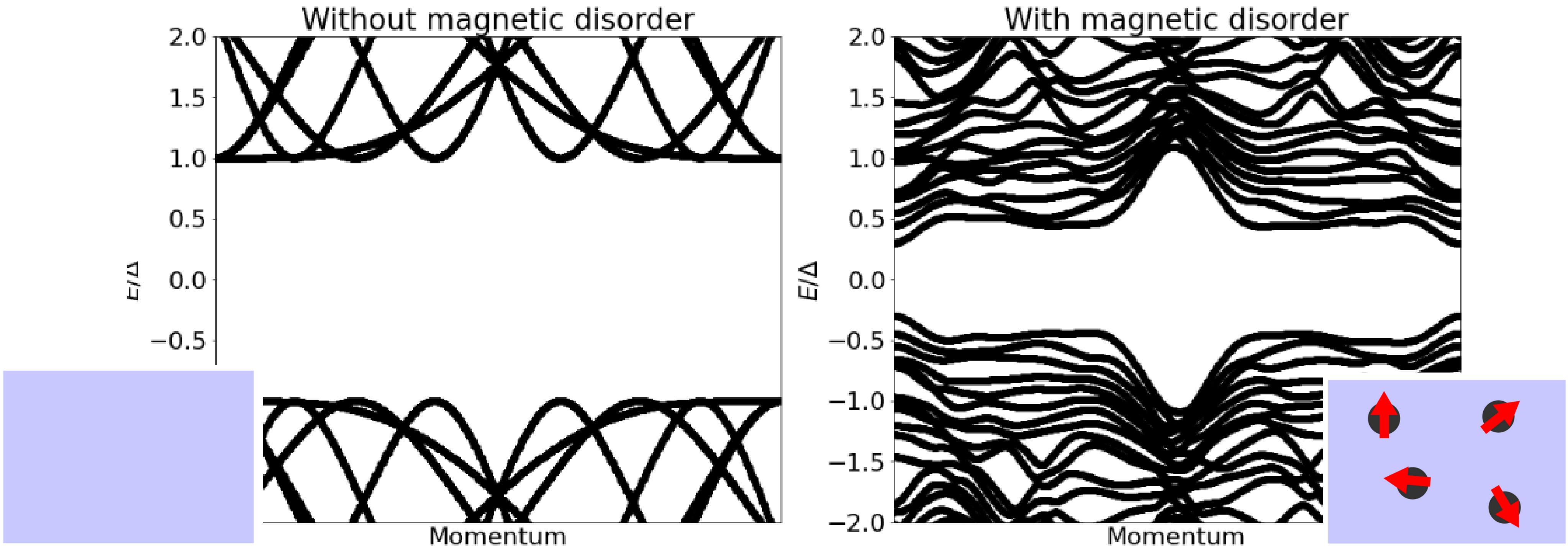
The interplay between magnetism and superconductivity

The exchange coupling controls the energy of the in-gap state



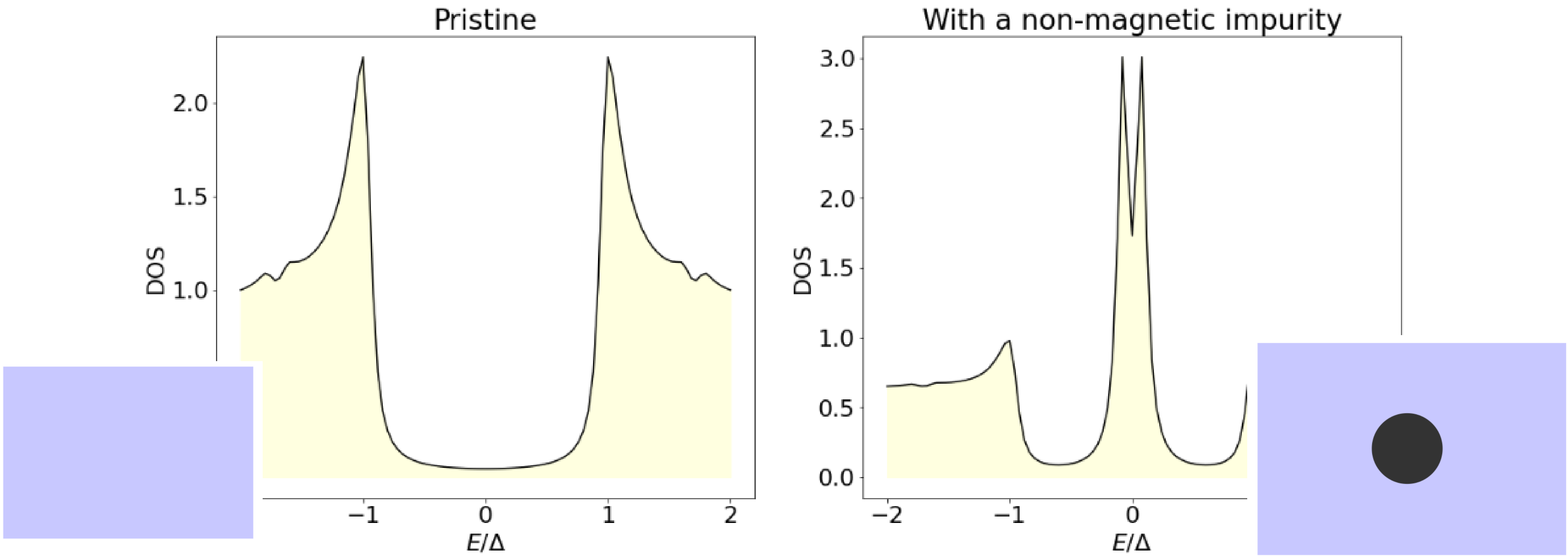


The effect of magnetic disorder in superconductors



Magnetic disorder decreases the gap of conventional superconductors

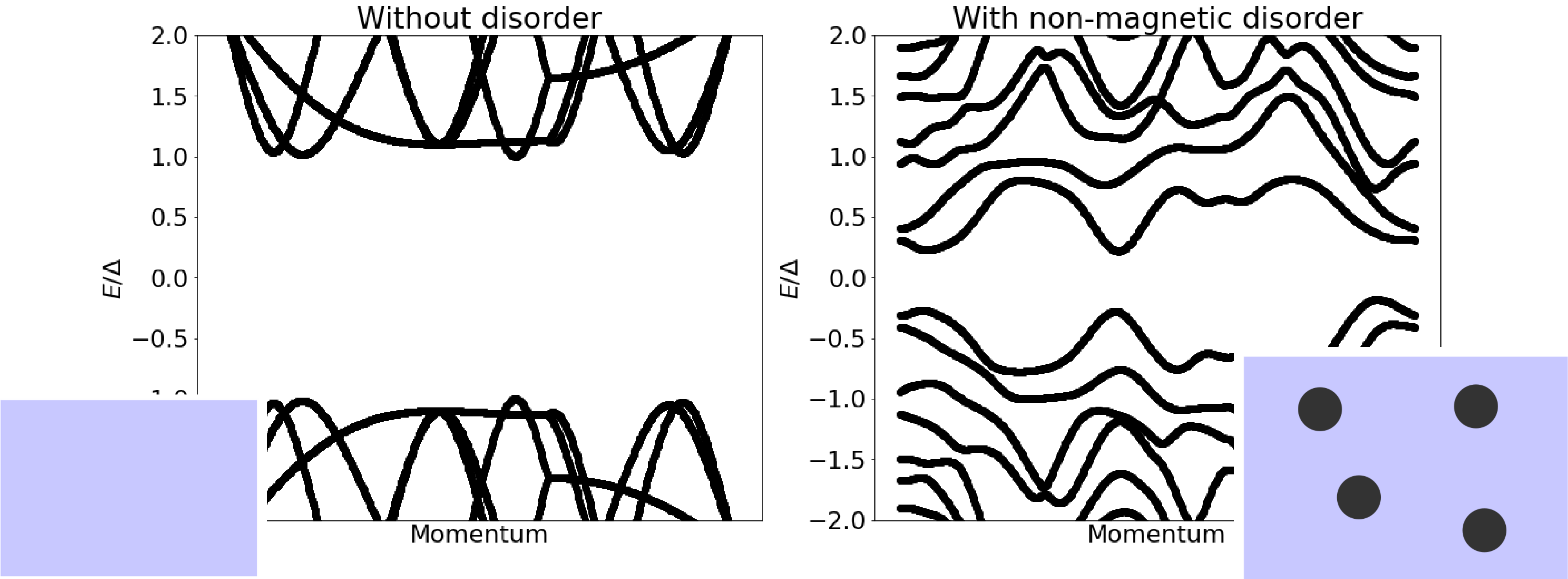
The interplay between impurities and unconventional superconductivity



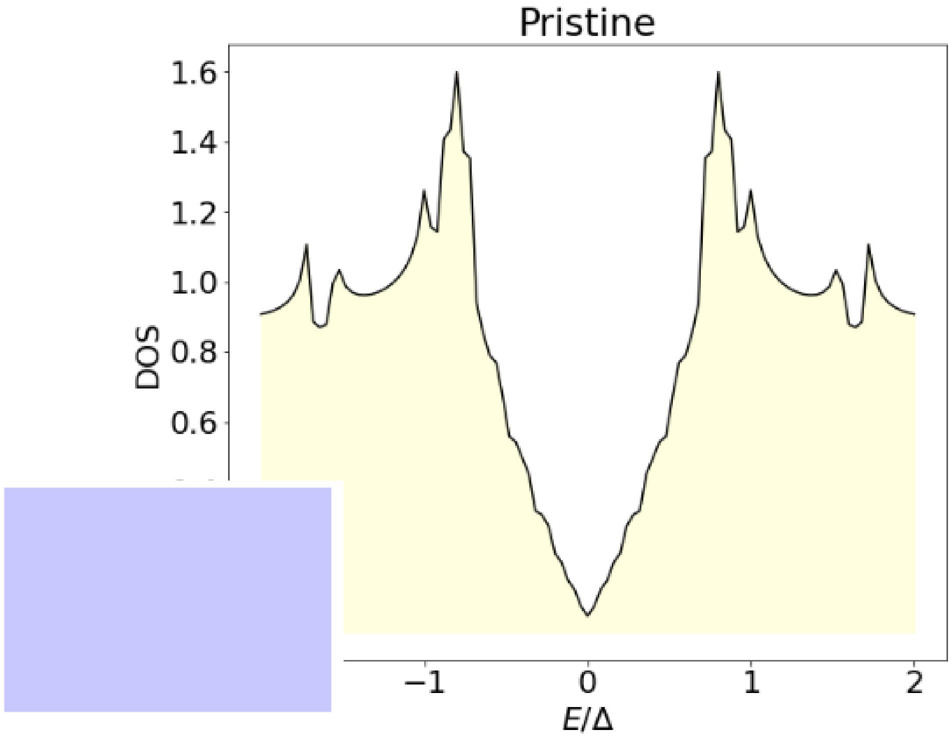
Non-magnetic impurities create in-gap states in fully gapped unconventional superconductors

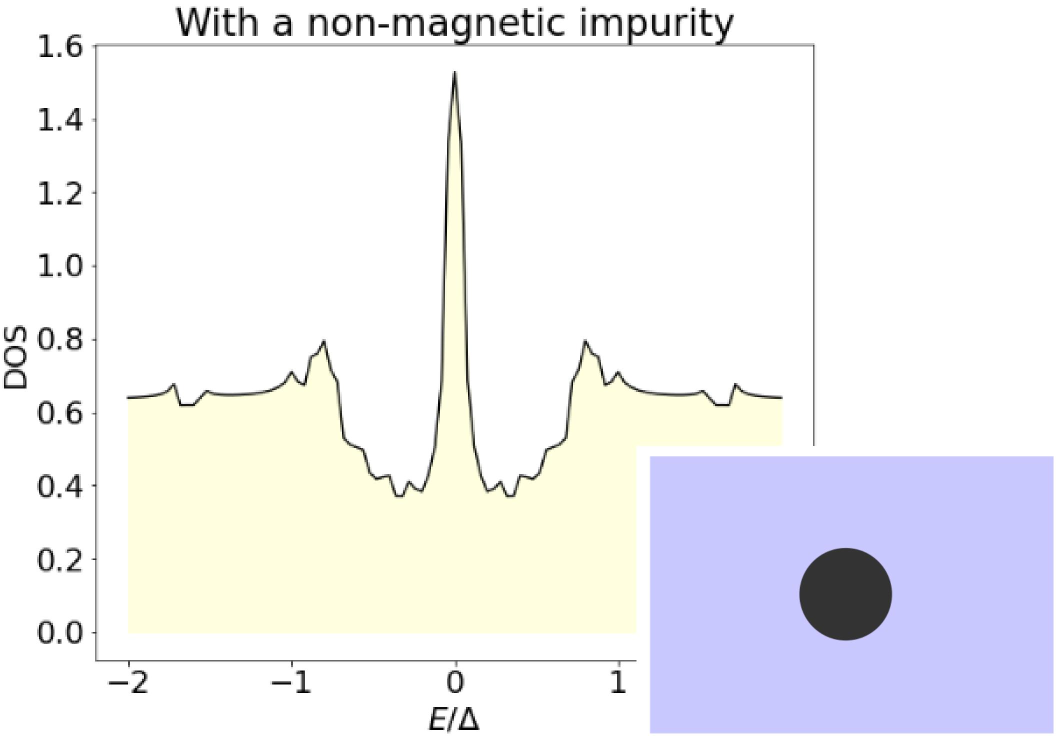
The impact of non-magnetic disorder in unconventional superconductors

Non-magnetic disorder in unconventional superconductors decreases the gap



Non-magnetic impurities in nodal superconductors





Non-magnetic impurities create in-gap states in nodal unconventional superconductors

Break

10-15 min break

(optional) to discuss during the break

What type of superconducting order is each one?

$$c^\dagger_{n,\uparrow} c^\dagger_{n,\downarrow} \qquad c^\dagger_{n,\uparrow} c^\dagger_{n+1,\downarrow} \qquad c^\dagger_{n,\uparrow} c^\dagger_{n+1,\uparrow}$$

and which symmetries do they break?

One-dimensional topological superconductivity

The Nambu representation

How do we solve a Hamiltonian of the form
$$H = \sum_{\mathbf{k},s} \epsilon_{\mathbf{k}} c^\dagger_{\mathbf{k},s} c_{\mathbf{k},s} + \sum_{\mathbf{k}} \Delta c^\dagger_{\mathbf{k},\uparrow} c^\dagger_{-\mathbf{k},\downarrow}$$

Define a Nambu spinor
$$\Psi_{\mathbf{k}} = \begin{pmatrix} c_{\mathbf{k}\uparrow} \\ c_{\mathbf{k}\downarrow} \\ c^\dagger_{-\mathbf{k}\downarrow} \\ -c^\dagger_{-\mathbf{k}\uparrow} \end{pmatrix}$$

Electron sector

Hole sector

The Hamiltonian in the Nambu basis is quadratic and can be diagonalized

$$H = \Psi^\dagger_{\mathbf{k}} \mathcal{H} \Psi_{\mathbf{k}}$$

Majorana excitations

A very special type of fermion is a so-called Majorana fermion

$$\Psi^\dagger = \Psi$$

Which by definition it is its own antiparticle

Majorana fermion do not appear naturally in materials, as we only have electrons

Yet, mathematically, each electron can be written as two Majoranas

$$c = \Psi_\alpha + i\Psi_\beta \qquad c^\dagger = \Psi_\alpha - i\Psi_\beta \qquad \Psi_\alpha^\dagger = \Psi_\alpha \qquad \Psi_\beta^\dagger = \Psi_\beta$$

Can we isolate a single Majorana in a material?

Looking at Majorana excitations in superconductors

Excitations in superconductors are combinations of electrons and holes, for instance

$$\Psi \sim c_n + c_n^\dagger$$

But this excitation is by definition a Majorana fermion

$$\Psi^\dagger = \Psi$$

Can we have superconductors in nature that show these excitations?

The minimal model for a 1D topological superconductor

One dimensional spinless p-wave superconductor (Kitaev model)

$$H = \sum_n t c^\dagger_{n+1} c_n + \Delta c_n c_{n+1} + c.c.$$

p-wave superconductivity

$$c_k = \sum_n e^{ikn} c_n$$

$$\Delta(k) = -\Delta(-k)$$

$$H = \sum_k \epsilon_k c^\dagger_k c_k + i\Delta \left[c_{-k} c_k \sin k - c^\dagger_{-k} c^\dagger_k \sin k \right]$$

A model Hamiltonian for topological superconductivity

Spinless fermions in a 1D chain (Kitaev model)

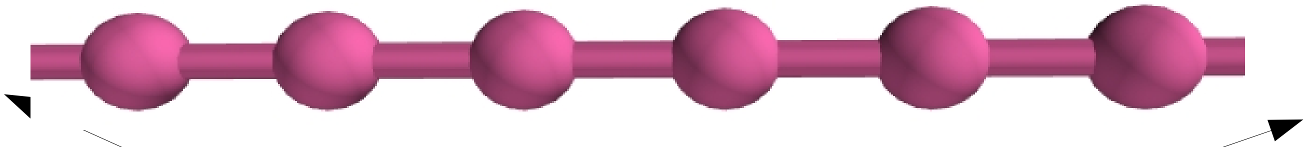
$$H = \sum_n c_n^\dagger c_{n+1} + c_n c_{n+1} + h.c.$$

$$c_n = \gamma_{2n-1} + i\gamma_{2n}$$

Can be transformed into

$$H = i\sum_n \gamma_{2n}\gamma_{2n+1}$$

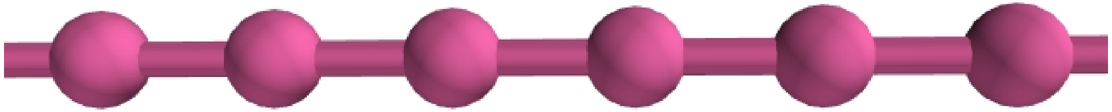
γ **Majorana operators**



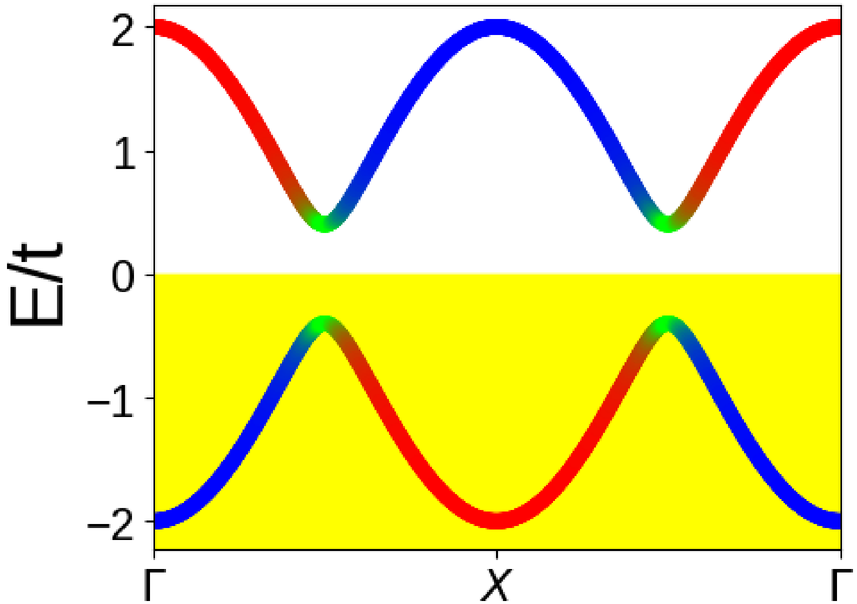
Unpaired Majorana operators

A model Hamiltonian for topological superconductivity

$$H = \sum_n c_n^\dagger c_{n+1} + \Delta c_n c_{n+1} + h.c.$$

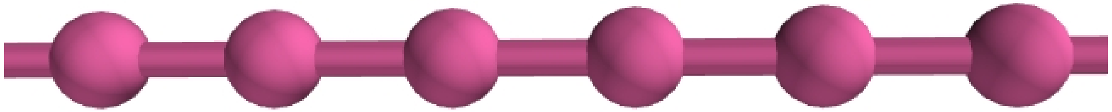


Infinite one dimensional chain

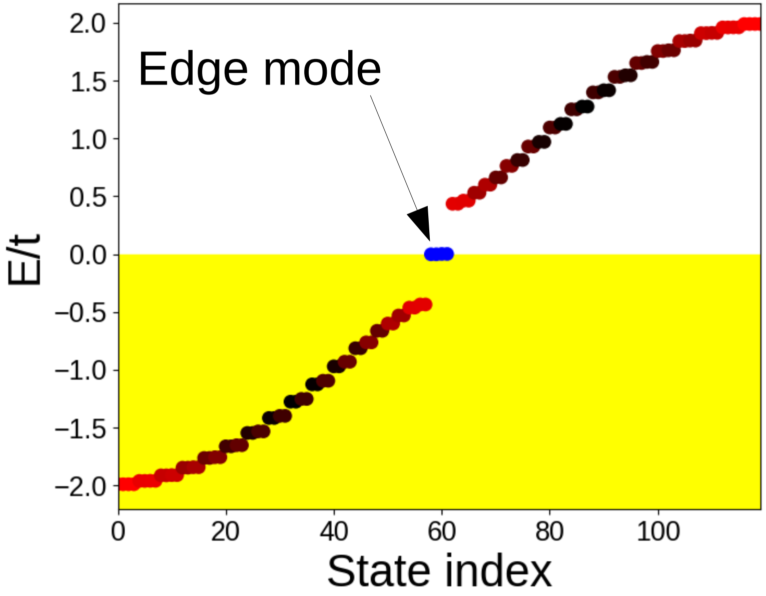


A model Hamiltonian for topological superconductivity

$$H = \sum_n c_n^\dagger c_{n+1} + \Delta c_n c_{n+1} + h.c.$$



Finite one dimensional chain



For finite systems, topological superconductivity gives rise to zero modes

Majorana states as topological surface modes

Generalized Kitaev model

$$H = \sum_n c_n^\dagger c_{n+1} + \Delta c_n c_{n+1} + \mu c_n^\dagger c_n + h.c.$$

p-wave SC

Chemical potential

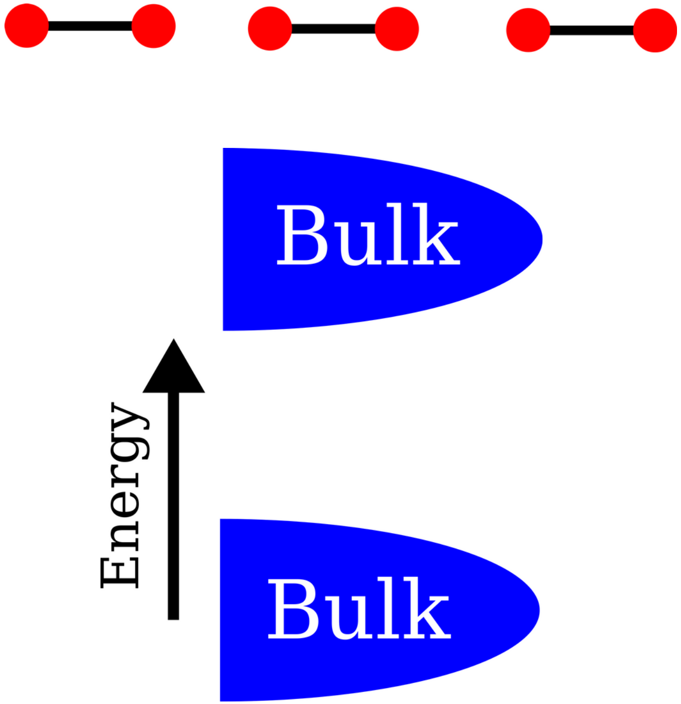
- Large chemical potential render the system filled, and topologically trivial
- p-wave SC promotes a topological phase with Majorana states

The emergence of Majorana states is associated to non-trivial topology

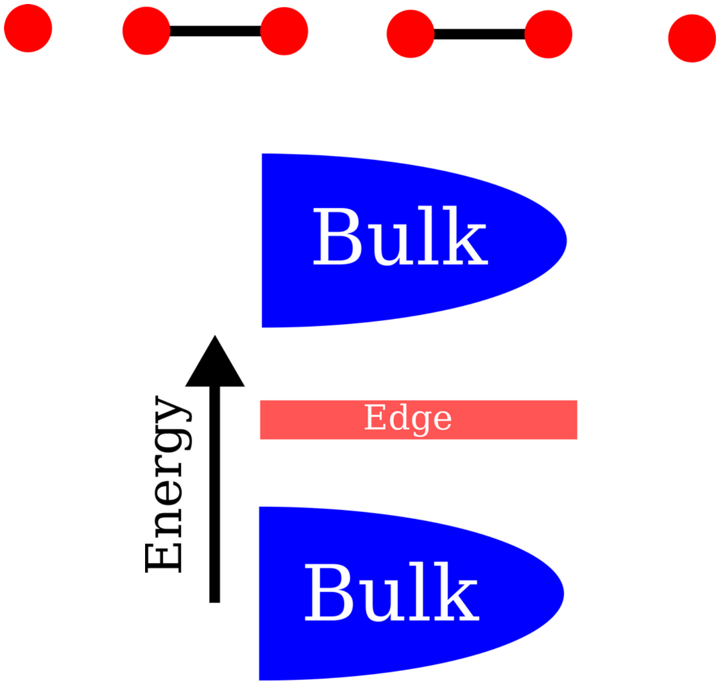
The two phases of the Kitaev model

(in the Majorana representation)

Trivial

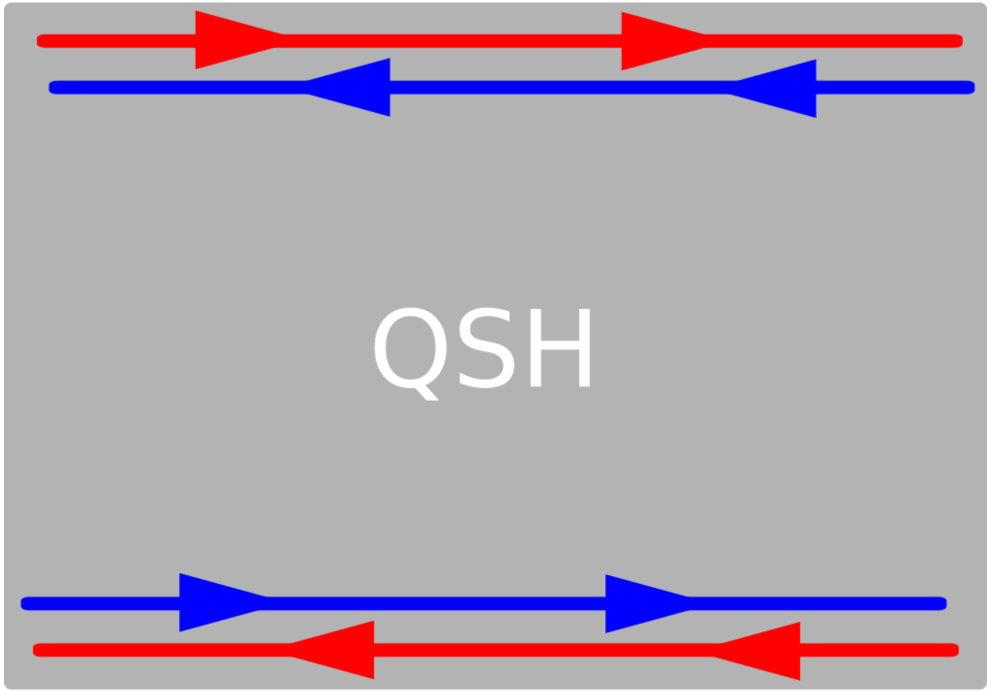


Topological



A simple way of building a topological superconductor with Majorana modes

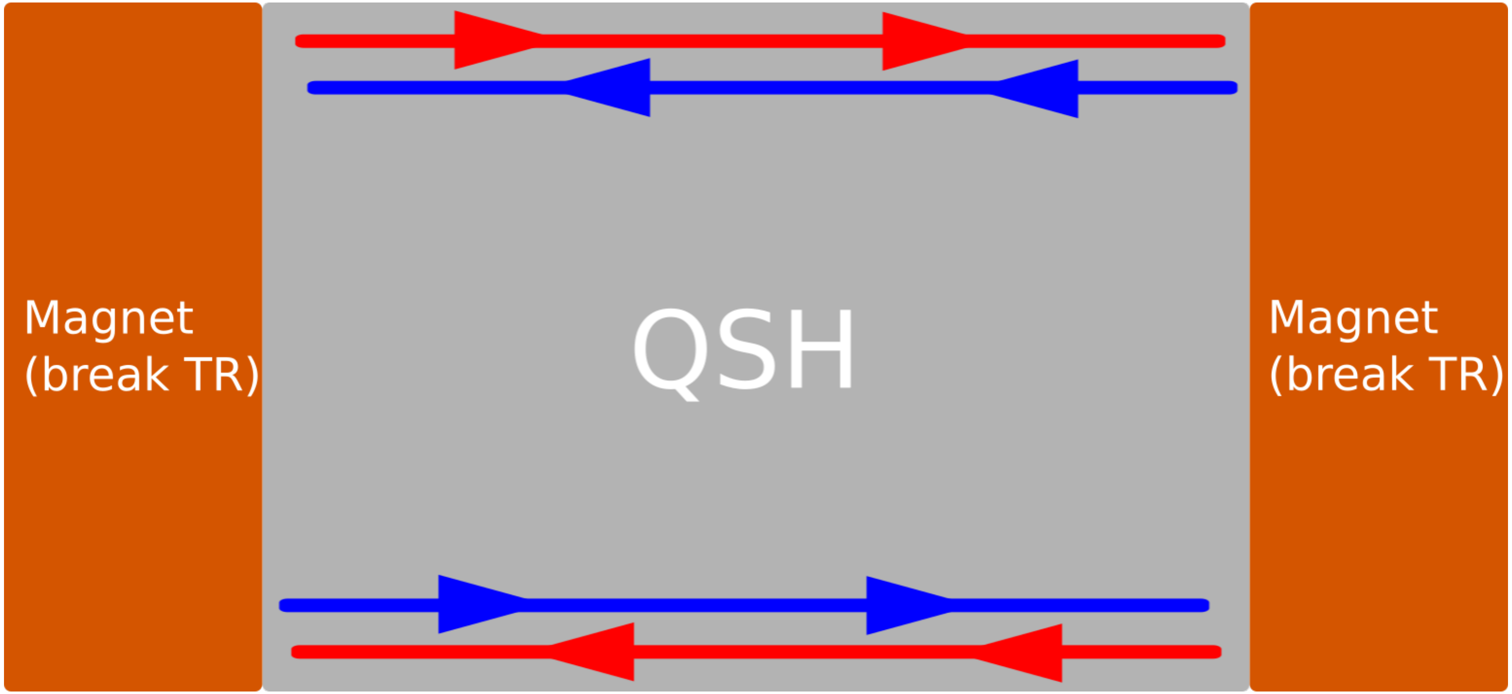
Taking the surface states of a quantum spin-Hall (QSH) insulator



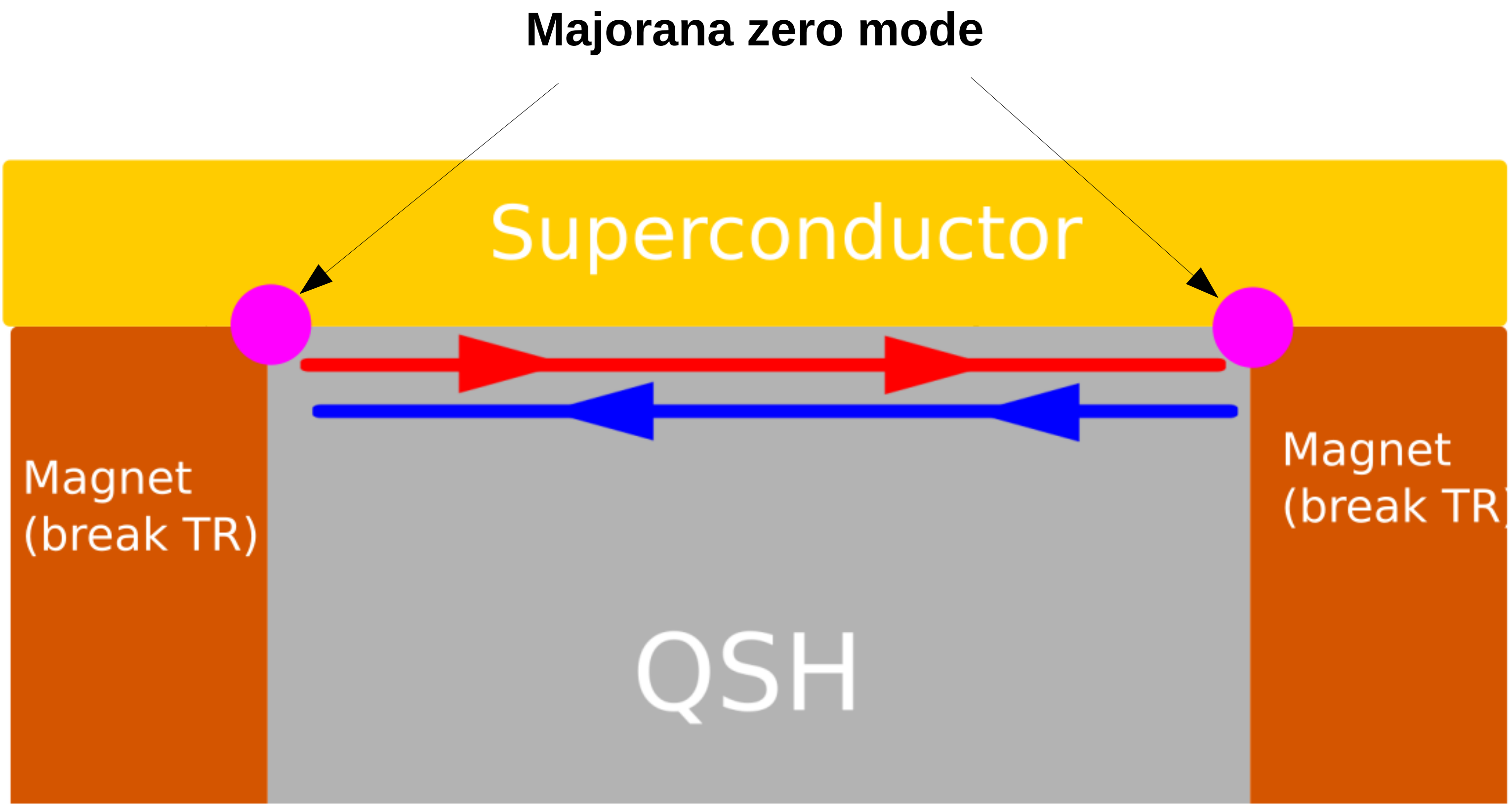
WTe_2

A simple way of building a topological superconductor with Majorana modes

Gap some of the helical modes with a magnet



A simple way of building a topological superconductor with Majorana modes



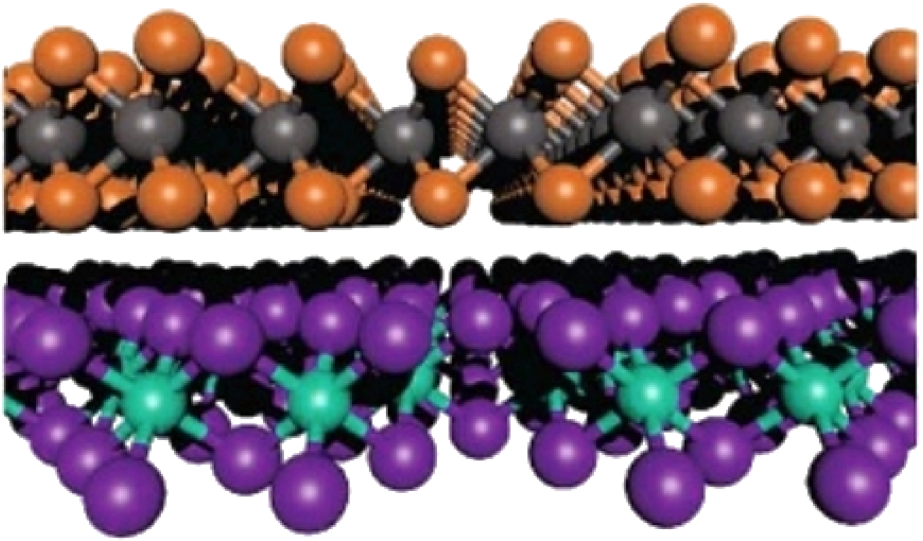
Two-dimensional artificial topological superconductivity

Engineering unconventional superconductors with conventional ones

$CrBr_3/NbSe_2$

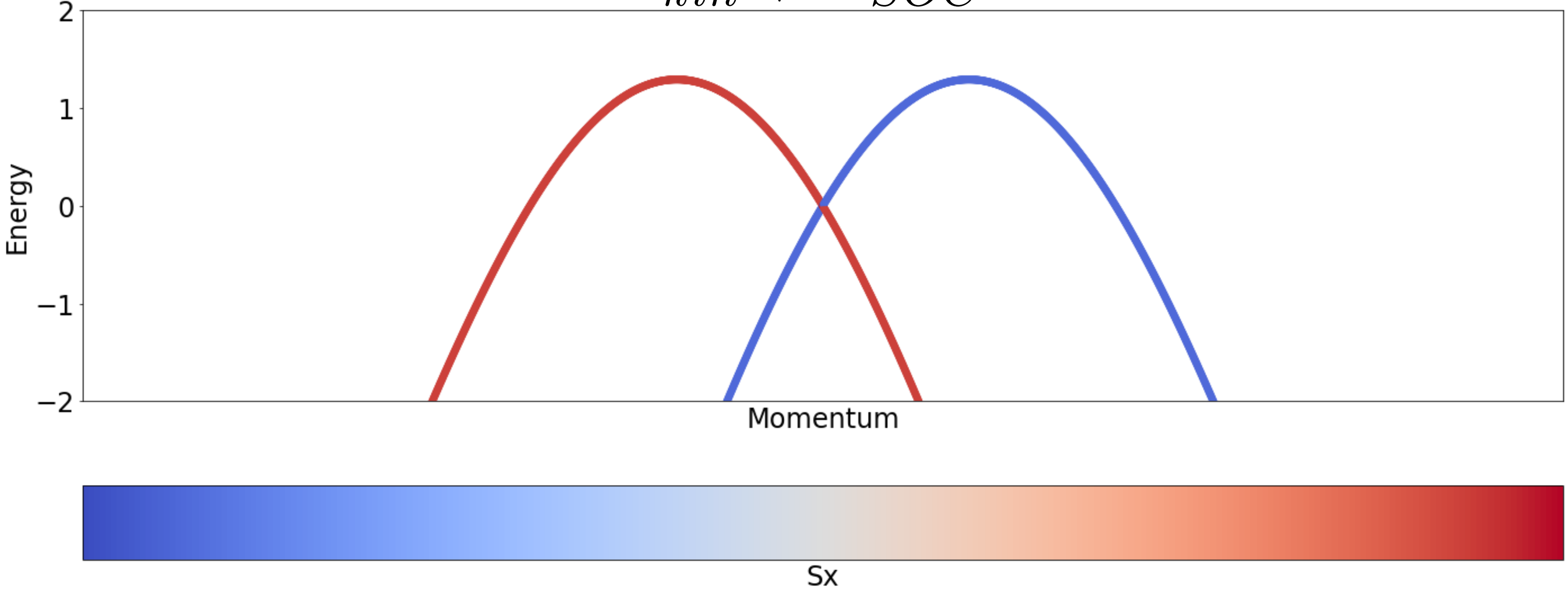
Van der Waals superconductor

Van der Waals ferromagnet



Engineering helical states

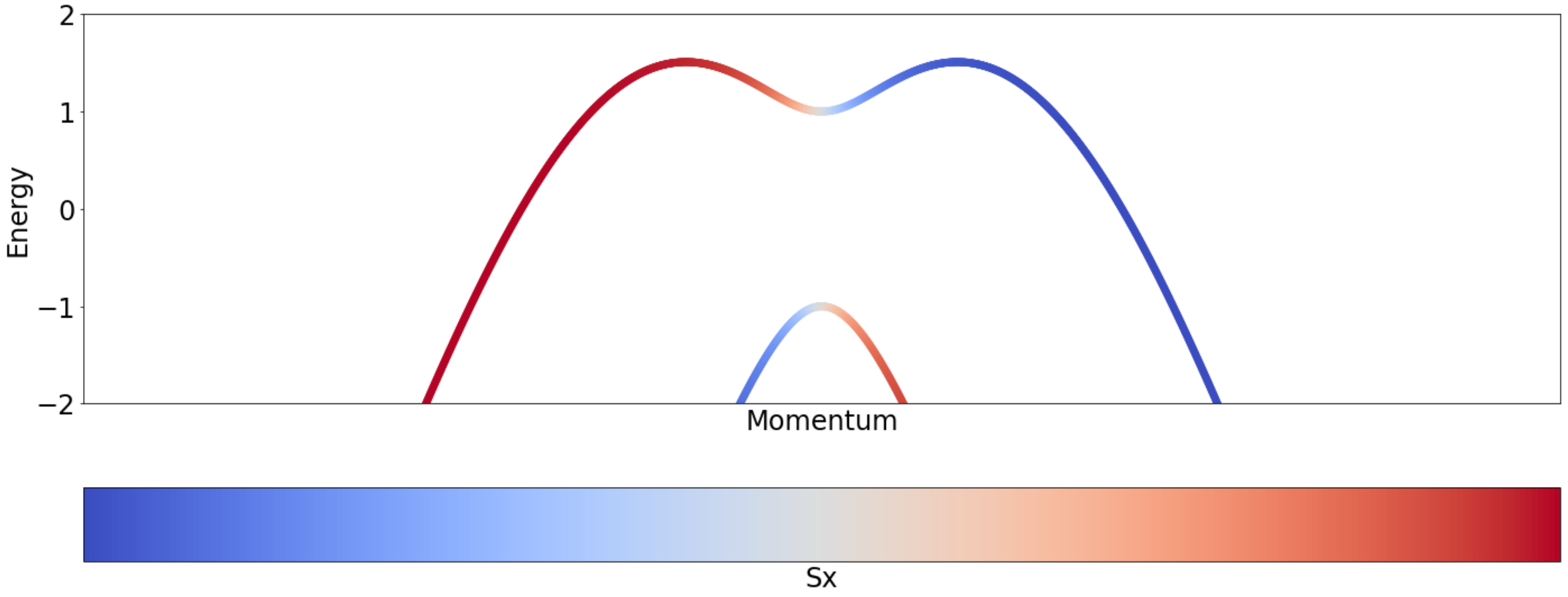
$$H_{kin} + H_{SOC}$$



With Rashba SOC, a spin-dependent spin splitting appears

Engineering helical states

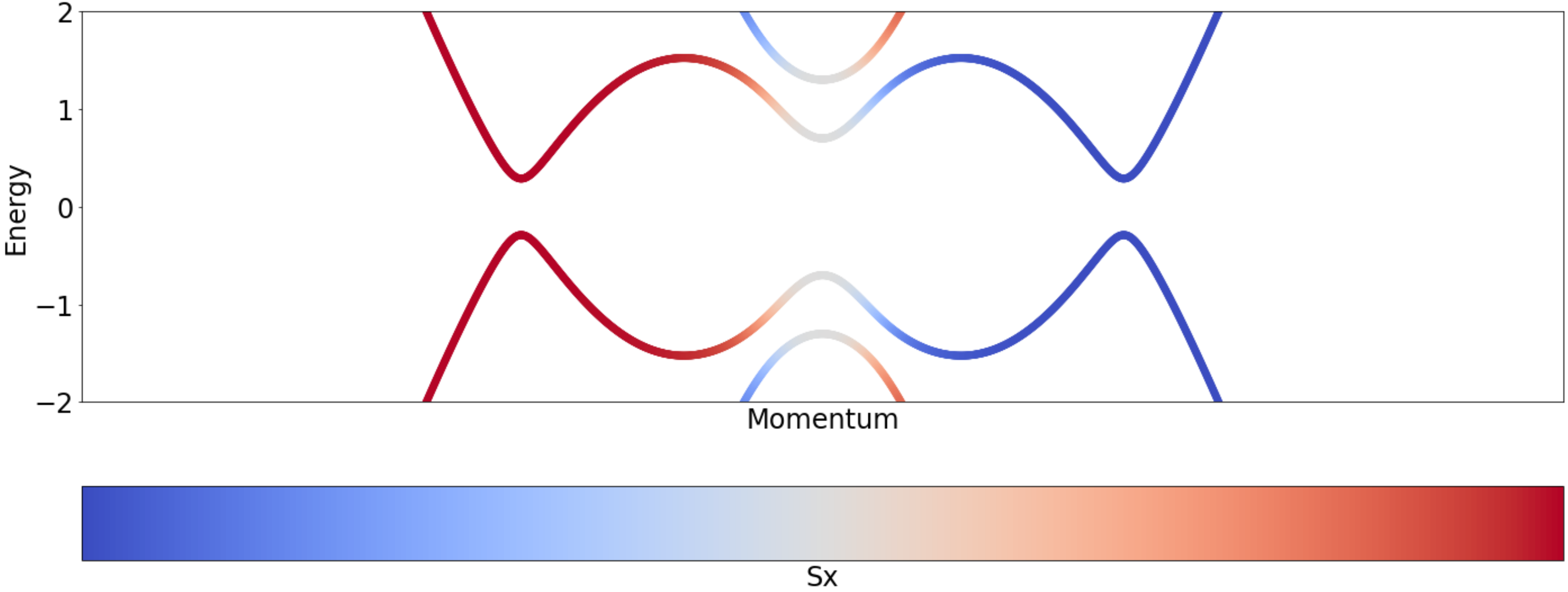
$$H_{kin} + H_J + H_{SOC}$$



With Rashba SOC and exchange, helical states appear

Engineering a topological superconductor

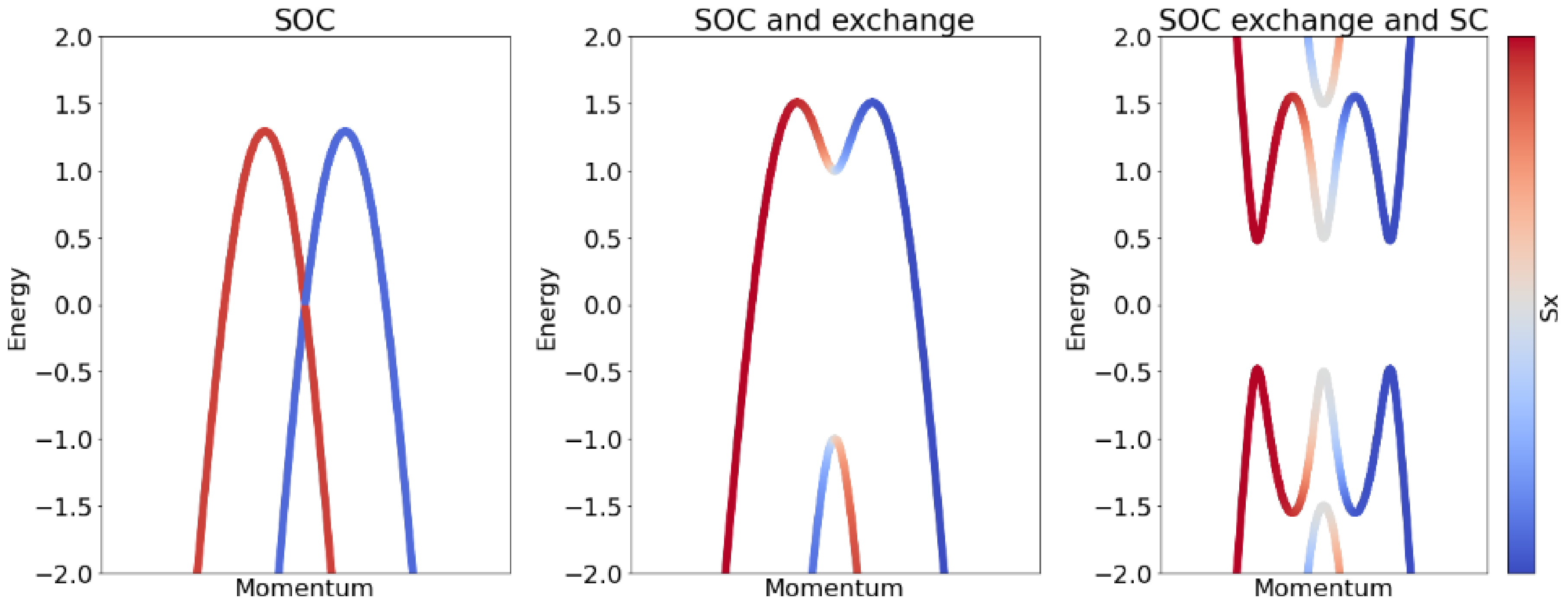
$$H_{kin} + H_J + H_{SOC} + H_{SC}$$



An s-wave superconducting gap opens up a tropological gap in the heterostructure

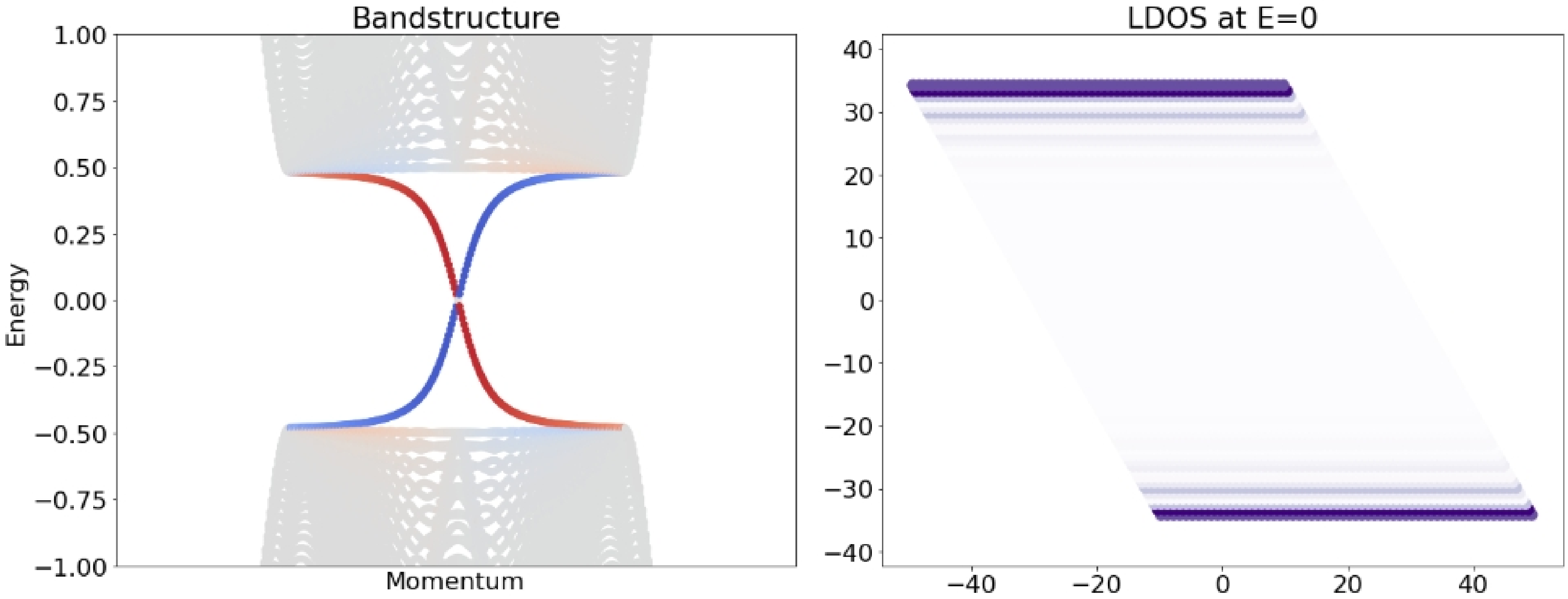
Artificial topological superconductivity C=1, bulk electronic structure

Bulk electronic structure



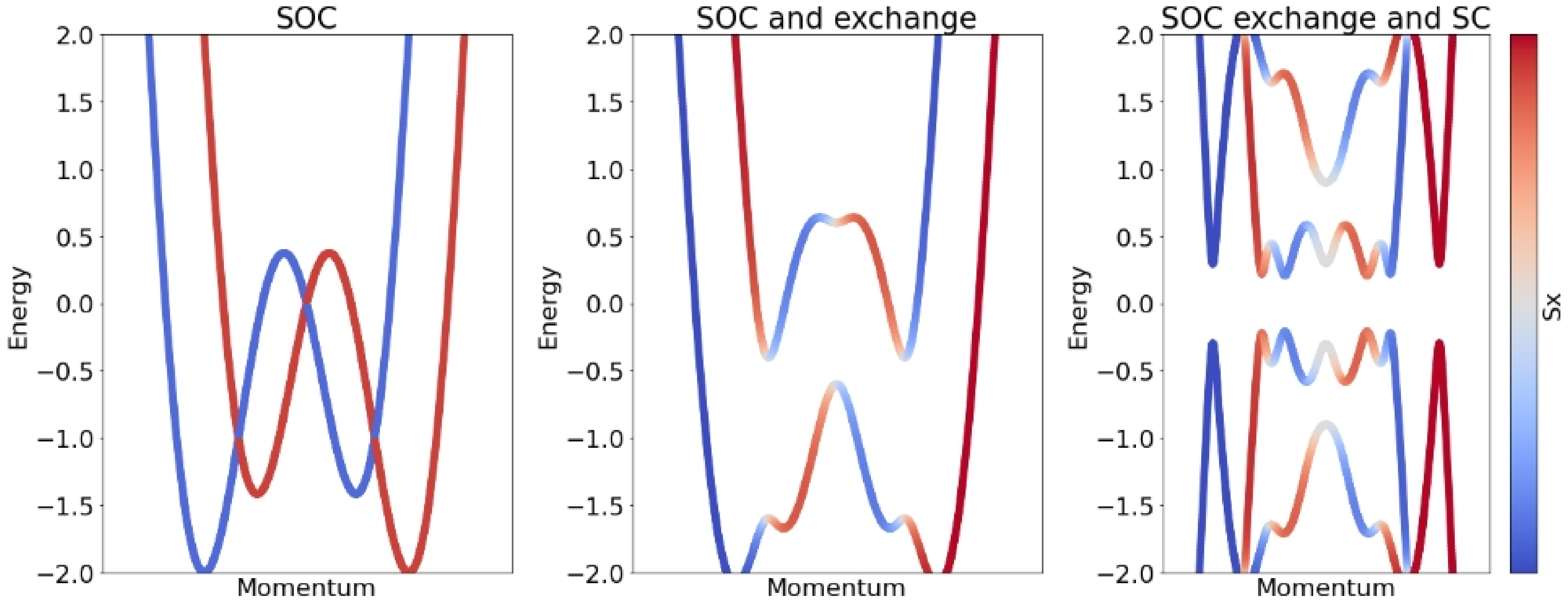
The combination of SOC and exchange creates helical states
Superconductivity gaps out the helical states in a non-trivial way

Artificial topological superconductivity C=1, ribbon electronic structure



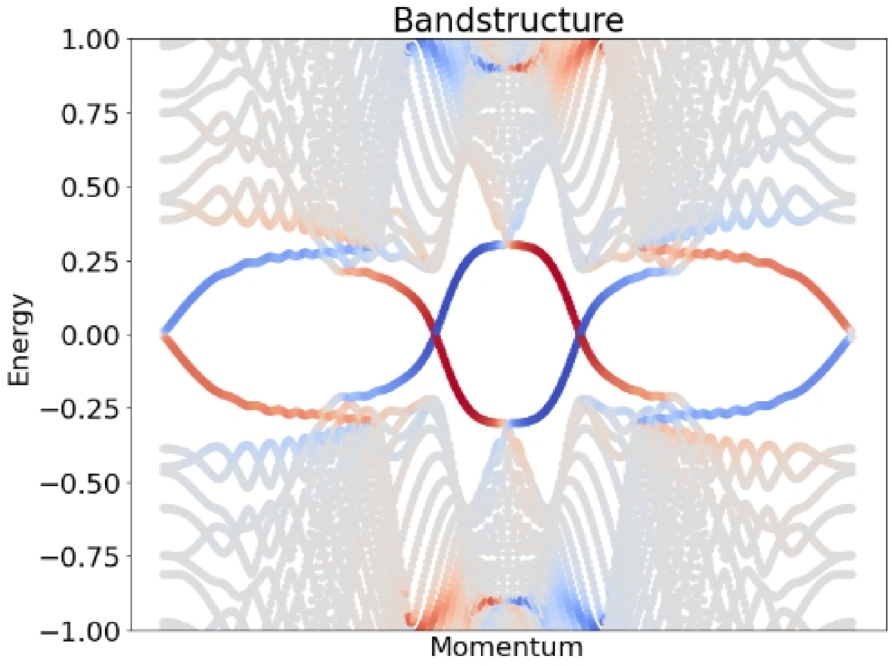
Artificial topological superconductivity C=3, bulk electronic structure

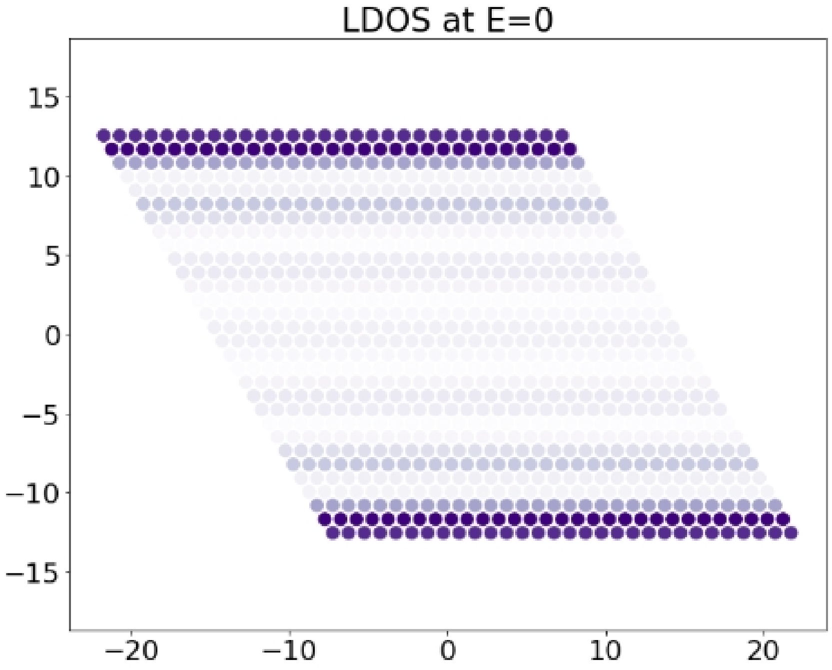
Bulk electronic structure



The combination of SOC and exchange creates helical states
Superconductivity gaps out the helical states in a non-trivial way

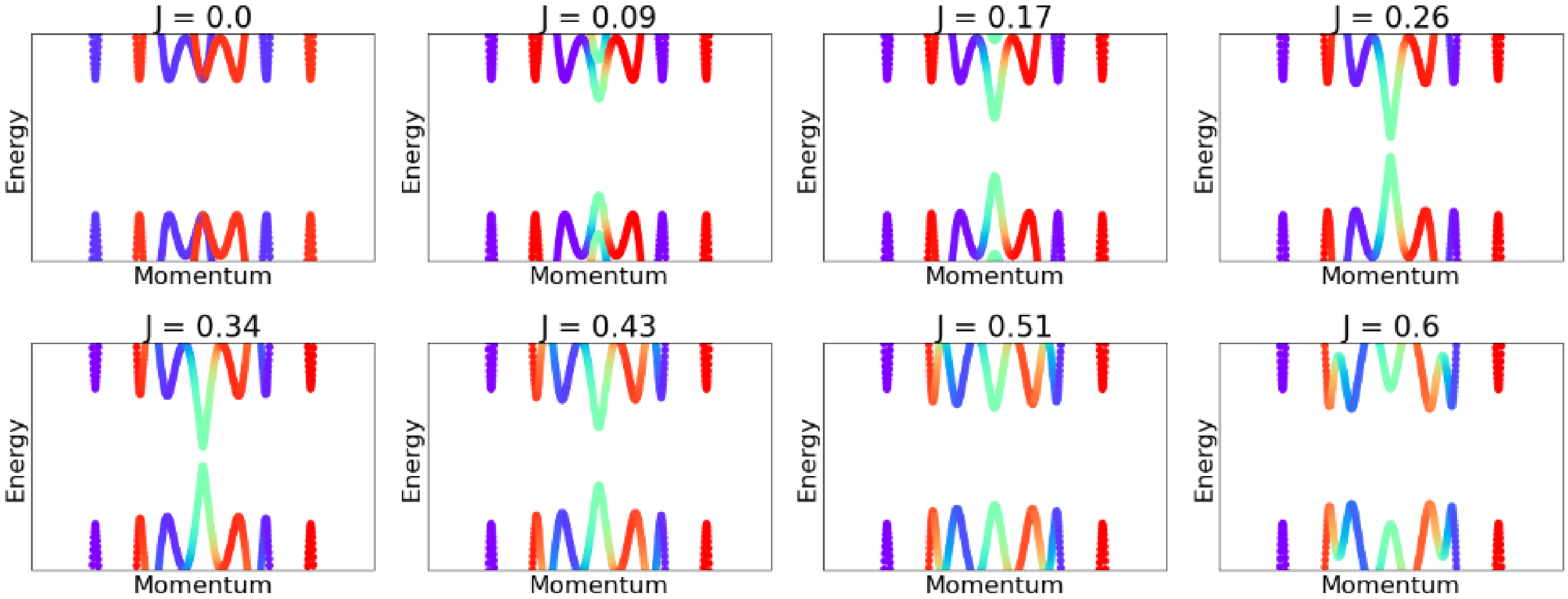
Artificial topological superconductivity C=3, ribbon electronic structure





Topological phase transition in the bulk

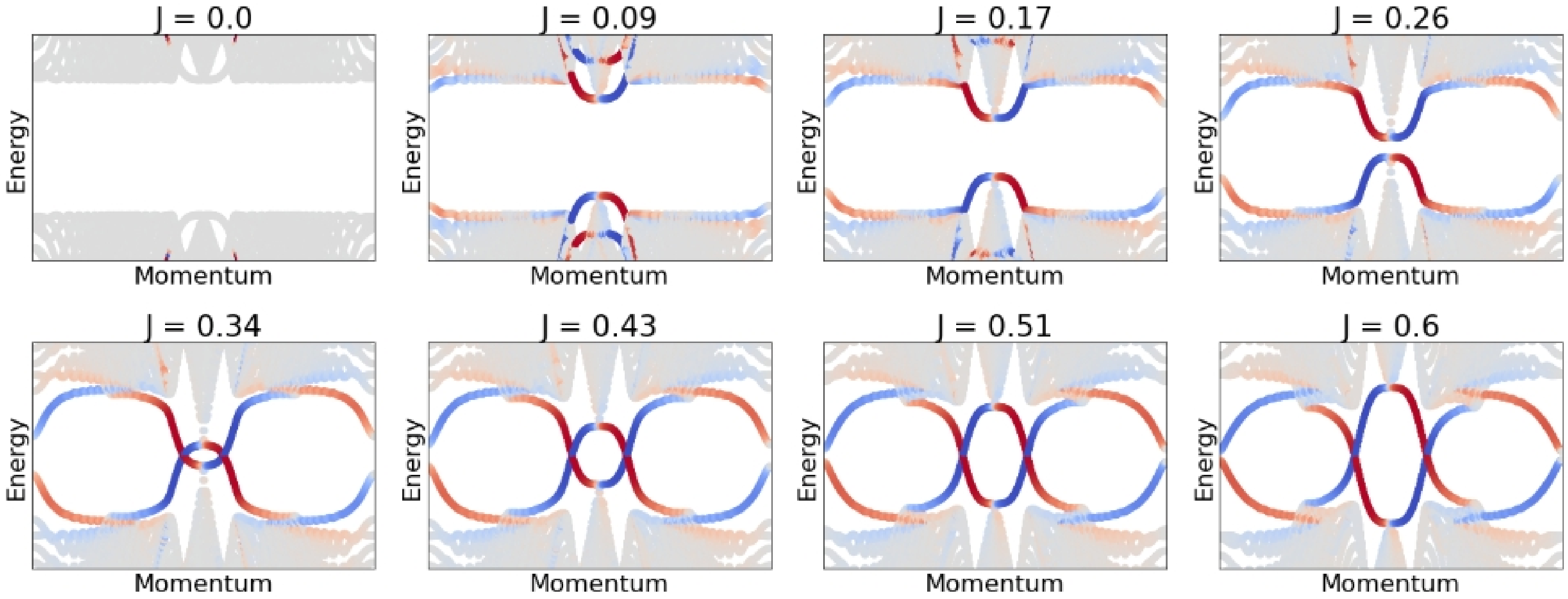
As we the value of the exchange coupling is changed,
a transition from trivial to topological emerges



A gap closing appears at a critical value of the exchange field

Topological phase transition in the edge

As we the value of the exchange coupling is changed,
a transition from trivial to topological emerges



A gap closing (and edge states) appear at a critical value of the exchange field

For the exercise session this afternoon

Download the Jupyter-notebook from

https://github.com/joselado/jyvaskyla_summer_school_2022/blob/main/sessions/session2.ipynb

The tasks during the exercise sessions

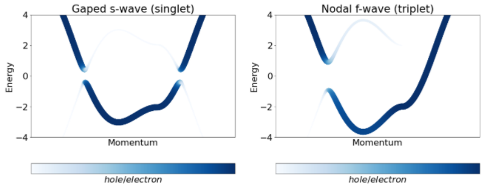
You will see examples with the code

```
from pyqula import geometry
g = geometry.triangular_lattice() # generate a triangular lattice

# first s-wave
plt.subplot(1,2,1) ;  plt.title("Gaped s-wave (singlet)")
h = g.get_hamiltonian() # generate Hamiltonian
h.add_pairing(mode="swave",delta=0.4) # add swave pairing
(k,e,c) = h.get_bands(operator="electron",kpath=["G","K","M","G"]) # compute bands
plt.scatter(k,e,c=c,s=200*c,cmap="Blues") ; plt.xlabel("Momentum")
plt.ylabel("Energy") ; plt.xticks([]) ; plt.ylim([-4,4]) # plot
plt.colorbar(label="$hole/electron$",ticks=[],orientation="horizontal")

# now nodal f-wave
plt.subplot(1,2,2) ;  plt.title("Nodal f-wave (triplet)")
h = g.get_hamiltonian() # generate Hamiltonian
h.add_pairing(mode="nodal_fwave",delta=0.2) # add swave pairing
(k,e,c) = h.get_bands(operator="electron",kpath=["G","K","M","G"]) # compute bands
plt.scatter(k,e,c=c,s=200*c,cmap="Blues") ; plt.xlabel("Momentum")
plt.ylabel("Energy") ; plt.xticks([]) ; plt.ylim([-4,4]) # plot
plt.colorbar(label="$hole/electron$",ticks=[],orientation="horizontal")
```

```
<matplotlib.colorbar.Colorbar at 0x7f515456a4c0>
```



You have to modify them, and answer questions

Exercise

- Discuss why an f-wave order is a triplet superconducting order parameter
- Discuss in which points of the Brillouin zone the triplet order (odd order parameter) must have nodes